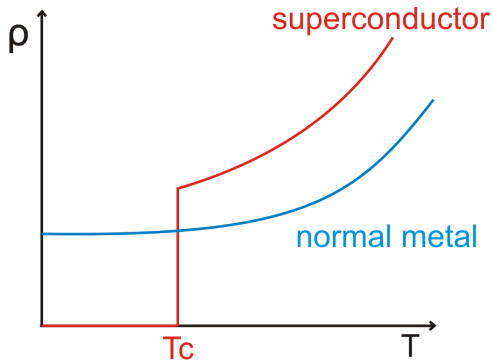


## Superconductivity

### Reading:

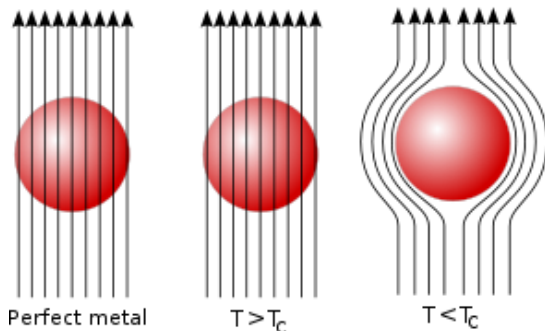
- ▶ Kasap: 8.9 - 8.10
- ▶ *Nature* **518**, 179 (2015) for high  $T_c$

## Zero resistance



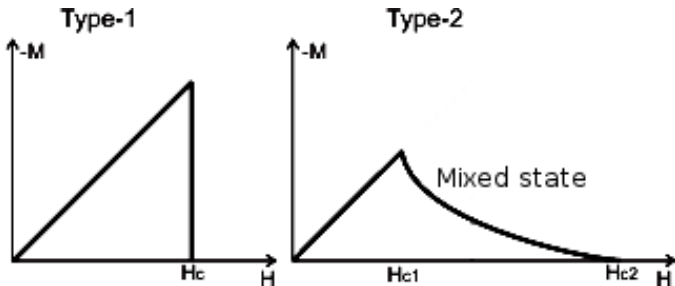
- ▶ Many metals exhibit zero resistance below a temperature  $T_c$
- ▶ First discovered in 1911 in mercury ( $T_c = 4.2$  K)
- ▶ Poorer conductors are usually better superconductors (higher  $T_c$ )
- ▶ Gold, copper and silver have negligible  $T_c$
- ▶ Highest among pure metals: lead ( $T_c = 7.2$  K) and niobium ( $T_c = 9.3$  K)

## Meissner effect



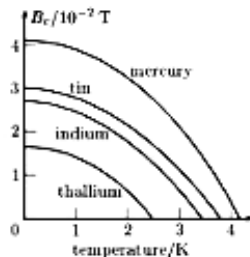
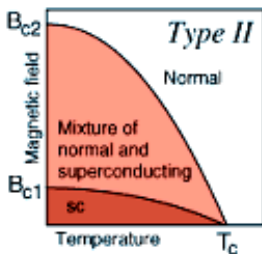
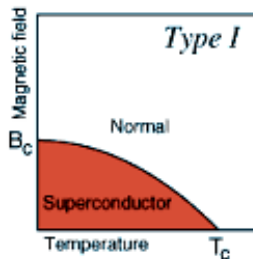
- ▶ Superconductor  $\neq$  perfect conductor
- ▶ Perfect conductor permits magnetic fields (like normal metal)
- ▶ Superconductor expels magnetic field completely
- ▶ Meissner effect: perfect diamagnetism  $\chi_m = -1$
- ▶ Typical paramagnetic or diamagnetic metals,  $|\chi_m| \sim 10^{-6} - 10^{-4}$

## Critical fields



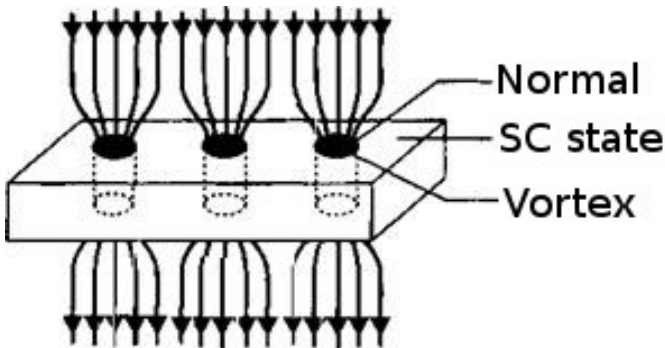
- ▶ Perfect diamagnet up to a maximum field  $H_c$
- ▶ Two types of behavior possible beyond  $H_c$
- ▶ Type I: Abrupt transition to normal state  $\rho \neq 0$ ,  $\chi_m \sim 0$
- ▶ Type II: gradual reduction of  $|\chi_m| \rightarrow 0$  (with  $\rho = 0$ )
- ▶ Mixed state of  $\rho = 0$  and  $\chi_m > -1$  ranges from lower critical field  $H_{c1}$  to upper critical field  $H_{c2}$

## Critical fields: $T$ dependence



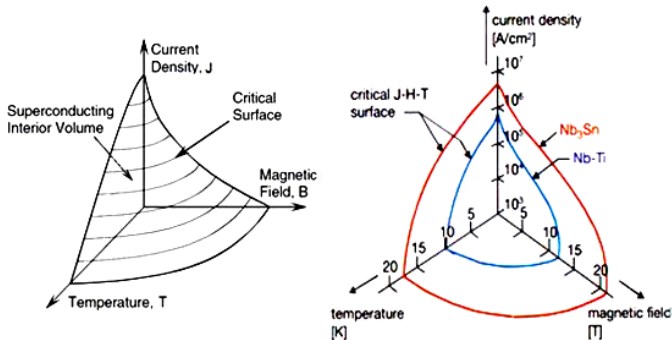
- ▶ Critical field decreases with temperature (exactly like ferromagnets)
- ▶ Higher critical field correlated with higher  $T_c$
- ▶ Upper and lower critical fields vanish together at  $T_c$  for type II (no  $T_{c1}$  and  $T_{c2}$ )

## Mixed state: vortices



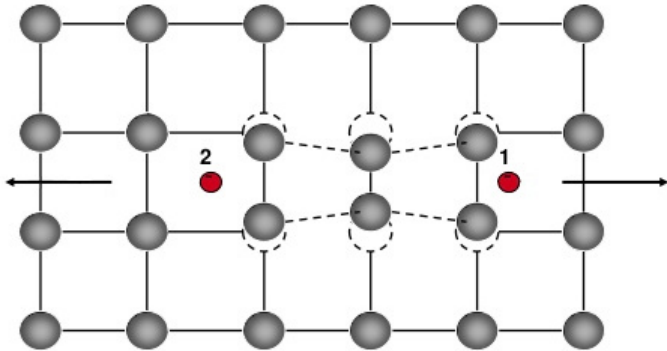
- ▶ Field enters superconductor in domains: vortices
- ▶ Normal state inside, and superconducting (SC) state outside
- ▶ Quantum of flux  $\Phi_0 = \frac{h}{2e} \approx 2 \times 10^{-15} \text{ Tm}^2$  in each vortex
- ▶ Increasing density of vortices between  $H_{c1}$  and  $H_{c2}$
- ▶ At  $H_{c2}$ , no SC region left  $\Rightarrow \rho \neq 0$

# Critical current



- ▶ Beyond current density  $j_c$ , switch to normal state
- ▶ Primary application of superconductors: high-field magnets
- ▶  $T$ ,  $j$  and  $H$  ( $B$ ) all push SC to normal state
- ▶ Limiting magnet performance limited by critical surface
- ▶ For metals and alloys,  $j_c$  and  $H_c$  increase with  $T_c$
- ▶ Not generally true for high- $T_c$  materials

## Microscopic origins: e-ph interactions



- ▶ One electron distorts the lattice (emits / absorbs a phonon)
- ▶ This distortion reduces potential for second electron
- ▶ Attractive electron-electron interaction mediated by phonons
- ▶ Attraction  $-\Delta$  between electrons at  $k \uparrow$  and  $-k \downarrow$
- ▶ All electrons near Fermi surface in Cooper pairs with energy reduced by  $\Delta$
- ▶ Result: band gap  $\Delta$  near Fermi surface: no free electrons



## Microscopic origins: BCS theory

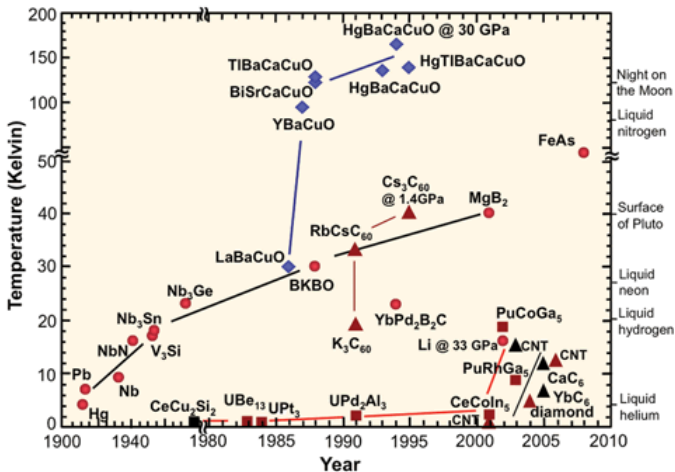
- ▶ Attraction produces band gap  $\Delta$
- ▶ But, pairs behave like spin zero particles with net charge  $2e$
- ▶ Bosons: all pairs in same quantum state for  $T < T_c$
- ▶ Apply field, all pairs carry current together (coherently)
- ▶ Gap  $\Delta \Rightarrow$  no states to scatter into!
- ▶ Perfect conductor *because* of gap!
- ▶ Facilitated by e-ph interactions  $\Rightarrow$  resistive metals are better superconductors!
- ▶ Increase temperature: pairs break thermally ( $T_c$ )
- ▶ Increase magnetic field: imbalance in spin energies breaks pair ( $H_c$ )
- ▶ Increase current density: pairs have enough momentum to scatter against electrons ( $j_c$ )

## Microscopic origins: Type I vs Type II

Two important length scales in superconductors

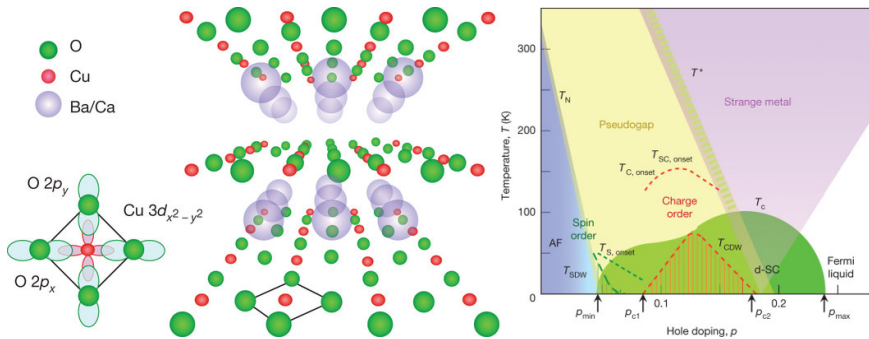
1. Coherence length  $\xi$ : length scale of the quantum wavefunction variation
2. Penetration depth  $\lambda$ : length scale of magnetic field variation
  - ▶ When  $\xi > \lambda$ , cost of breaking wavefunction higher:  
stay SC till pairs break (Type I)
  - ▶ When  $\xi < \lambda$ , break wavefunction to relax magnetic field energy  
 $\Rightarrow$  favorable to form vortices (Type II)
  - ▶ Vortex: normal state extent  $\sim \xi$  surrounded by field region  $\sim \lambda$

# $T_c$ versus time



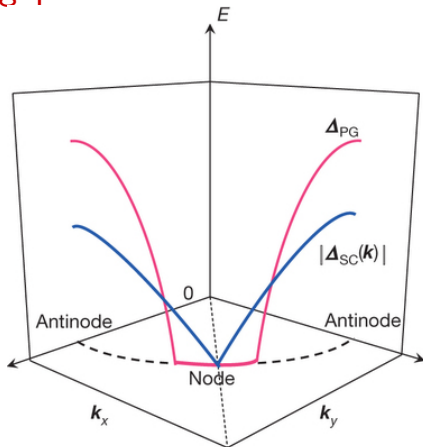
- ▶ Conventional metals with  $T_c < 10$  K and alloys with  $T_c < 40$  K
- ▶ New classes of materials with  $T_c$  approaching 160 K!

# Typical phase diagram of cuprate superconductors

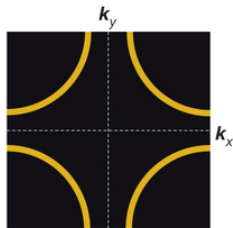
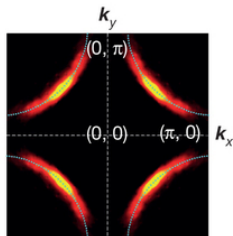


- ▶ Correlated-electron materials: many poorly understood phases!
- ▶ Antiferromagnetism, spin-density waves and strange metals
- ▶ Strange metal: resistivity  $\propto T$  even when  $\lambda < a$
- ▶  $d$ -wave superconductor: pairs have  $l = 2$  instead of  $l = 0$  (BCS)

## Pseudogap: Fermi surface arcs



$$\Delta_{SC}(\mathbf{k}) = (\Delta_0/2) [\cos(k_x a) - \cos(k_y a)]$$



- ▶ No real superconducting gap  $\Delta$
- ▶ Anisotropic gap in states with zero gap in certain directions
- ▶ Angular dependence of gap like a  $d$ -orbital
- ▶ Read *Nature* **518**, 179 (2015) for more!