

Superconductivity

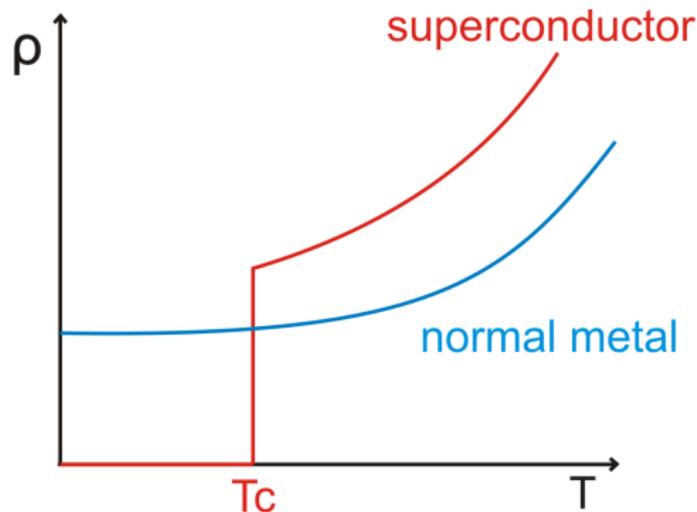
Contents:

- ▶ Phenomenology: resistance, Meissner effect, type I and II
- ▶ Microscopic origin of superconductivity
- ▶ High- T_c superconductivity

Reading:

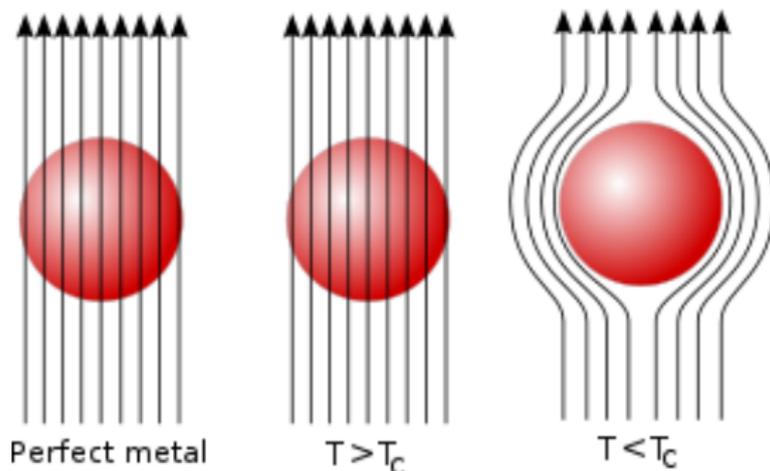
- ▶ Kasap: 8.9 - 8.10
- ▶ *Nature* **518**, 179 (2015) for high T_c

Zero resistance



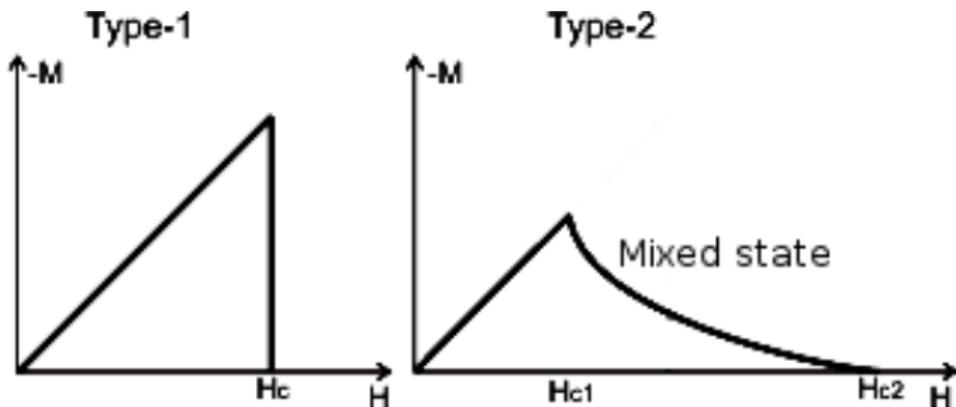
- ▶ Many metals exhibit zero resistance below a temperature T_c
- ▶ First discovered in 1911 in mercury ($T_c = 4.2$ K)
- ▶ Poorer conductors are usually better superconductors (higher T_c)
- ▶ Gold, copper and silver have negligible T_c
- ▶ Highest among pure metals: lead ($T_c = 7.2$ K) and niobium ($T_c = 9.3$ K)

Meissner effect



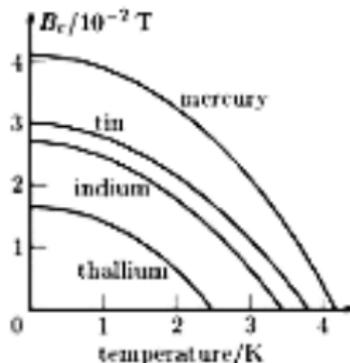
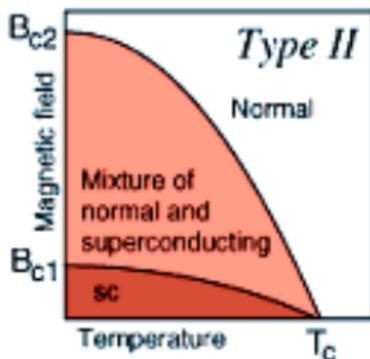
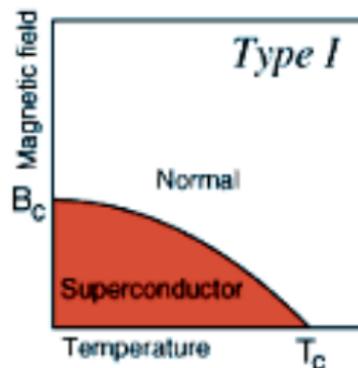
- ▶ Superconductor \neq perfect conductor
- ▶ Perfect conductor permits magnetic fields (like normal metal)
- ▶ Superconductor expels magnetic field completely
- ▶ Meissner effect: perfect diamagnetism $\chi_m = -1$
- ▶ Typical paramagnetic or diamagnetic metals, $|\chi_m| \sim 10^{-6} - 10^{-4}$

Critical fields



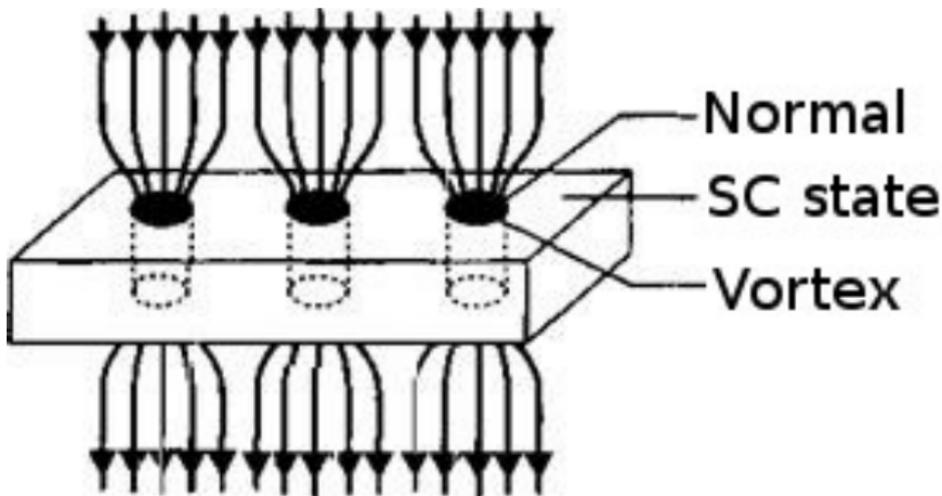
- ▶ Perfect diamagnet up to a maximum field H_c
- ▶ Two types of behavior possible beyond H_c
- ▶ Type I: Abrupt transition to normal state $\rho \neq 0$, $\chi_m \sim 0$
- ▶ Type II: gradual reduction of $|\chi_m| \rightarrow 0$ (with $\rho = 0$)
- ▶ Mixed state of $\rho = 0$ and $\chi_m > -1$ ranges from lower critical field H_{c1} to upper critical field H_{c2}

Critical fields: T dependence



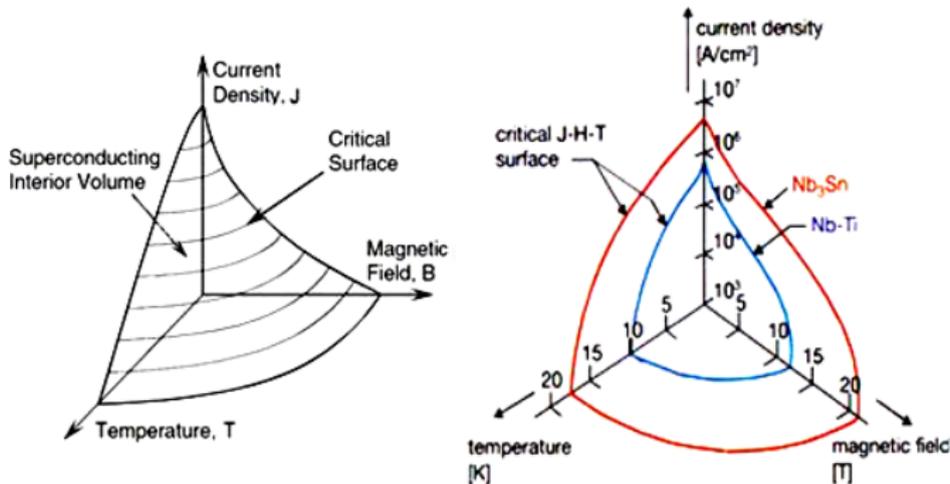
- ▶ Critical field decreases with temperature (exactly like ferromagnets)
- ▶ Higher critical field correlated with higher T_c
- ▶ Upper and lower critical fields vanish together at T_c for type II (no T_{c1} and T_{c2})

Mixed state: vortices



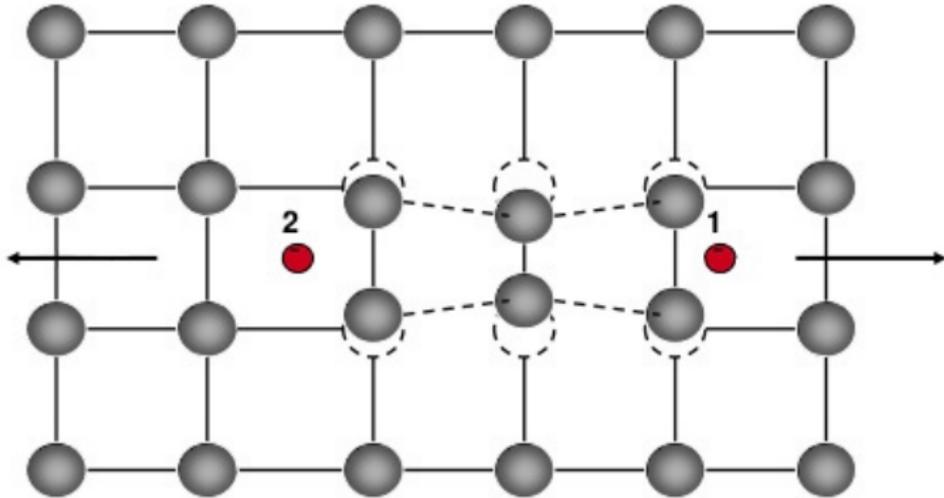
- ▶ Field enters superconductor in domains: vortices
- ▶ Normal state inside, and superconducting (SC) state outside
- ▶ Quantum of flux $\Phi_0 = \frac{h}{2e} \approx 2 \times 10^{-15} \text{ Tm}^2$ in each vortex
- ▶ Increasing density of vortices between H_{c1} and H_{c2}
- ▶ At H_{c2} , no SC region left $\Rightarrow \rho \neq 0$

Critical current



- ▶ Beyond current density j_c , switch to normal state
- ▶ Primary application of superconductors: high-field magnets
- ▶ T , j and H (B) all push SC to normal state
- ▶ Limiting magnet performance limited by critical surface
- ▶ For metals and alloys, j_c and H_c increase with T_c
- ▶ Not generally true for high- T_c materials

Microscopic origins: e-ph interactions



- ▶ One electron distorts the lattice (emits / absorbs a phonon)
- ▶ This distortion reduces potential for second electron
- ▶ Attractive electron-electron interaction mediated by phonons
- ▶ Attraction $-\Delta$ between electrons at $k \uparrow$ and $-k \downarrow$
- ▶ All electrons near Fermi surface in Cooper pairs with energy reduced by Δ
- ▶ Result: band gap Δ near Fermi surface: no free electrons

Microscopic origins: BCS theory

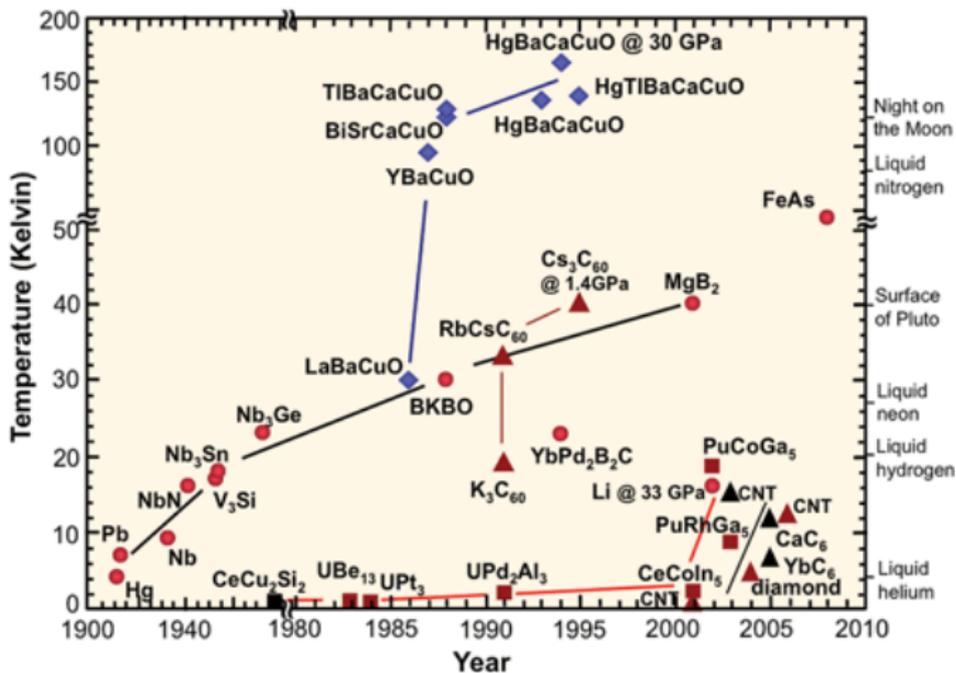
- ▶ Attraction produces band gap Δ
- ▶ But, pairs behave like spin zero particles with net charge $2e$
- ▶ Bosons: all pairs in same quantum state for $T < T_c$
- ▶ Apply field, all pairs carry current together (coherently)
- ▶ Gap $\Delta \Rightarrow$ no states to scatter into!
- ▶ Perfect conductor *because* of gap!
- ▶ Facilitated by e-ph interactions \Rightarrow resistive metals are better superconductors!
- ▶ Increase temperature: pairs break thermally (T_c)
- ▶ Increase magnetic field: imbalance in spin energies breaks pair (H_c)
- ▶ Increase current density: pairs have enough momentum to scatter against electrons (j_c)

Microscopic origins: Type I vs Type II

Two important length scales in superconductors

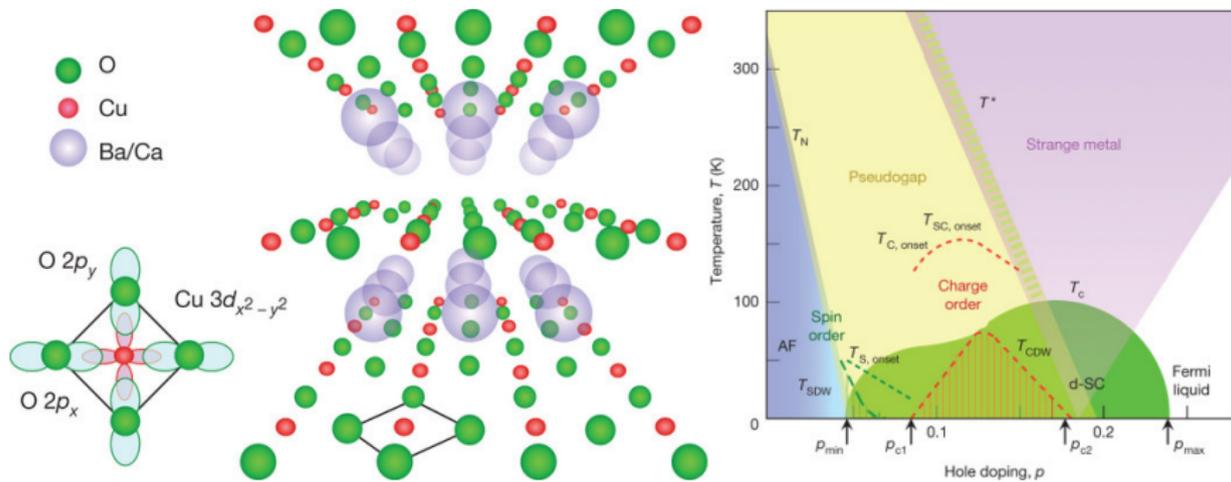
1. Coherence length ξ : length scale of the quantum wavefunction variation
 2. Penetration depth λ : length scale of magnetic field variation
- ▶ When $\xi > \lambda$, cost of breaking wavefunction higher:
stay SC till pairs break (Type I)
 - ▶ When $\xi < \lambda$, break wavefunction to relax magnetic field energy
 \Rightarrow favorable to form vortices (Type II)
 - ▶ Vortex: normal state extent $\sim \xi$ surrounded by field region $\sim \lambda$

T_c versus time



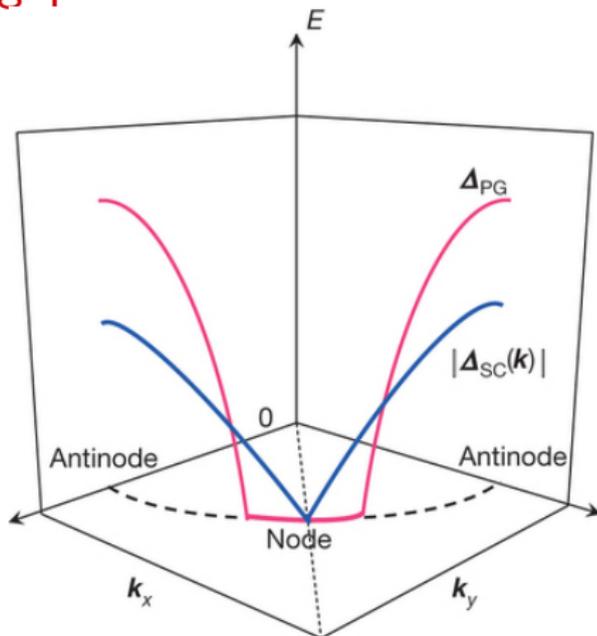
- ▶ Conventional metals with $T_c < 10$ K and alloys with $T_c < 40$ K
- ▶ New classes of materials with T_c approaching 160 K!

Typical phase diagram of cuprate superconductors

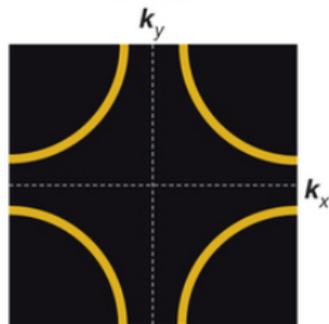
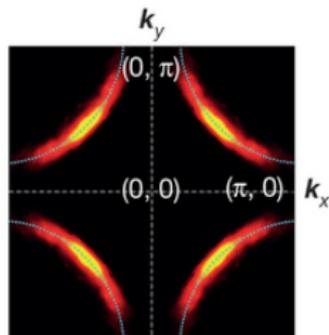


- ▶ Correlated-electron materials: many poorly understood phases!
- ▶ Antiferromagnetism, spin-density waves and strange metals
- ▶ Strange metal: resistivity $\propto T$ even when $\lambda < a$
- ▶ d -wave superconductor: pairs have $l = 2$ instead of $l = 0$ (BCS)

Pseudogap: Fermi surface arcs



$$\Delta_{SC}(\mathbf{k}) = (\Delta_0/2) [\cos(k_x a) - \cos(k_y a)]$$



- ▶ No real superconducting gap Δ
- ▶ Anisotropic gap in states with zero gap in certain directions
- ▶ Angular dependence of gap like a d -orbital
- ▶ Read *Nature* **518**, 179 (2015) for more!