

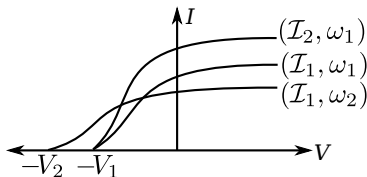
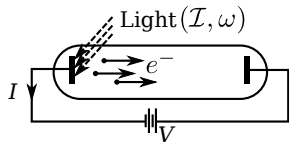
## Metal-vacuum junctions: thermal and field emission

### Reading:

- ▶ Kasap 4.9
- ▶ Review Kasap 3.1.2 (Photoelectric effect)

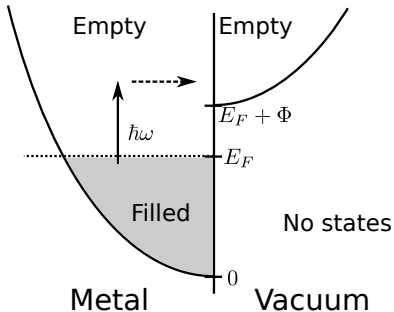
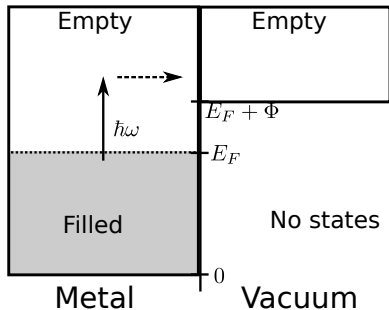
## Photoelectric effect

- ▶ Light ejects electrons from cathode  $\Rightarrow I$  at  $V = 0$
- ▶  $V \uparrow \Rightarrow I \uparrow$  till saturation (all ejected electrons collected)
- ▶  $V \downarrow \Rightarrow I \downarrow$  till  $I = 0$ :  
all electrons stopped at  $V = -V_0$
- ▶ Increase intensity  $\mathcal{I}$ :  
higher saturation  $I$  but same stopping  $V$
- ▶ Increase frequency  $\omega$ :  
higher stopping  $V$
- ▶ Stopping action:  $eV_0 = KE_{\max}$
- ▶ Experiment finds  $eV_0 \propto (\omega - \omega_0)$
- ▶ In fact  $eV_0 = \hbar(\omega - \omega_0)$
- ▶ Different cathodes  $\Rightarrow$  different  $\omega_0$   
but same slope  $\hbar$  identical to that  
from Planck's law!
- ▶ Light waves with angular frequency  $\omega$  behave like  
particles (photons) with energy  $\hbar\omega$  (Einstein, 1905)



## Workfunction: energy level alignment with vacuum

- ▶ Minimum energy  $\Phi$  required to free electron from material
- ▶ Photoelectric effect threshold is  $\hbar\omega_0 = \Phi$
- ▶ Electrons emitted with kinetic energy  $\text{KE} = \hbar\omega - \hbar\omega_0$
- ▶ Determined by alignment of energy levels across metal-vacuum interface



## What determines workfunction?

- ▶ Electron binding in bulk material (strongly bound  $\Rightarrow$  higher  $\Phi$ )
- ▶ Equally important: surface of the metal i.e. metal-vacuum interface
- ▶ Energy-level alignment sensitive to details of the surface
- ▶ Example: work functions (in eV) of single crystalline metal surfaces

Metal	(110)	(100)	(111)	Polycrystalline
Al	4.06	4.20	4.26	4.1 – 4.3
Au	5.12	5.00	5.30	5.1 – 5.4
Ag	4.52	4.64	4.74	4.3 – 4.7
Cu	4.48	4.59	4.94	4.5 – 5.1

- ▶ Values for polycrystalline metals averaged over facets (whose relative prominence depends on sample preparation)

# Thermionic emission

- ▶ Overcome energy difference (barrier) using thermal energy
- ▶ Number of electrons above barrier:

$$\int_{E_F + \Phi}^{\infty} dE g(E) f(E) \approx \int_{E_F + \Phi}^{\infty} dE g(E) \exp \frac{-(E - E_F)}{k_B T}$$

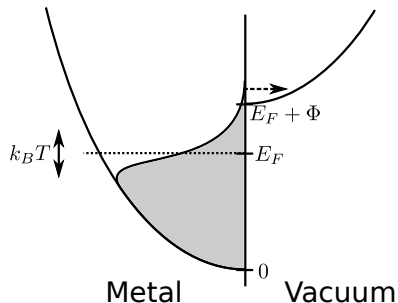
(assuming  $\Phi \gg k_B T$ , which holds for metals even at  $T_{\text{melt}}$ )

- ▶ Can all these electrons cross?
- ▶ Need KE towards surface

$$\frac{m(v \cos \theta)^2}{2} > E_F + \Phi$$

- ▶ Current density per state:

$$\langle ev \cos \theta \rangle = \frac{ev(1 - \frac{E_F + \Phi}{E})}{4}$$



## Richardson-Dushman equation

- ▶ Current density of emitted electrons:

$$j = \int_{E_F + \Phi}^{\infty} dE g(E) \exp \frac{-(E - E_F)}{k_B T} \cdot \frac{ev(1 - \frac{E_F + \Phi}{E})}{4}$$

- ▶ Assuming  $\Phi \gg k_B T$  and free-electron  $g(E) = 4\pi\sqrt{E} \left(\frac{\sqrt{2m}}{2\pi\hbar}\right)^3$ :

$$j = \underbrace{\frac{4\pi emk_B^2}{(2\pi\hbar)^3}}_{B_0} T^2 \exp \frac{-\Phi}{k_B T}$$

with Richardson-Dushman constant  $B_0 \approx 1.20 \times 10^6 \text{ A}/(\text{mK})^2$

- ▶ Additional consideration: electrons with sufficient KE can still be reflected
- ▶ Include energy-dependent reflection coefficient in above consideration
- ▶ Modified  $B_e \lesssim B_0/2$  for most metals,  $\ll B_0$  for some *d*-metals (why?)

## Electron near a metal surface

- ▶ Metal surface at constant potential; electric field normal
- ▶ Electric field outside as if due to charge and its reflection

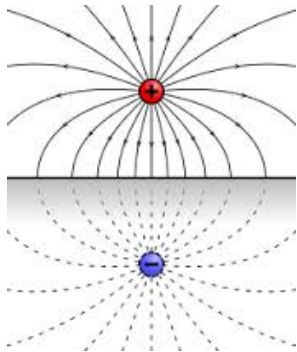
$$\vec{E}(\vec{r}) = \frac{q(\vec{r} - z\hat{z})}{4\pi\epsilon_0|\vec{r} - z\hat{z}|^3} - \frac{q(\vec{r} + z\hat{z})}{4\pi\epsilon_0|\vec{r} + z\hat{z}|^3}$$

- ▶ Force on charge:

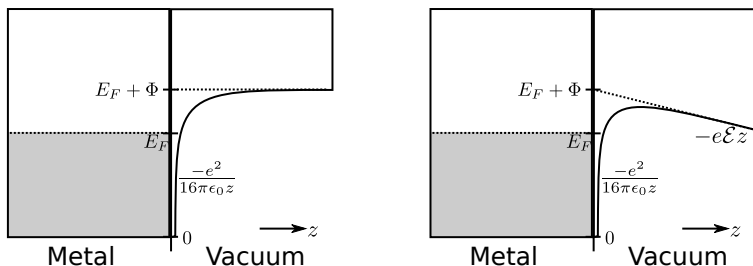
$$\vec{F} = \frac{-q^2\hat{z}}{4\pi\epsilon_0(2z)^2}$$

- ▶ Potential energy:

$$U = - \int_{\infty}^z \vec{F} \cdot \hat{z} = \frac{-q^2}{16\pi\epsilon_0 z}$$



# Schottky effect



- ▶ Image charge effect changes energy level diagram (horizontal axis is now distance from interface)
- ▶ What is the energy barrier for electrons at  $E_F$ ?
- ▶ Now consider an applied electric field  $\mathcal{E}$
- ▶ Net minimum energy level of electron is now:

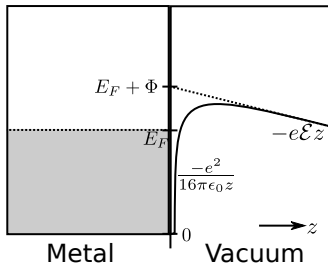
$$E_{\min}(z > 0) = E_F + \Phi - \frac{e^2}{16\pi\epsilon_0 z} - e\mathcal{E}z \leq E_F + \Phi - \sqrt{\frac{e^3\mathcal{E}}{16\pi\epsilon_0}}$$

- ▶ Barrier reduced to  $\Phi - \beta_s\sqrt{\mathcal{E}}$  with Schottky coefficient  $\beta_s = \sqrt{e^3/(16\pi\epsilon_0)} \approx 3.79 \times 10^{-5} \text{ eV}/\sqrt{\text{V/m}}$



## Field emission

- ▶ Electric field reduces effective barrier for electron emission
- ▶ Still use thermal energy, but with a lower barrier  $\Rightarrow$  use lower  $T$
- ▶ Technically field-assisted thermionic emission
- ▶ Use sharpened metal tips / nanowires / nanotubes to enhance local  $\mathcal{E}$
- ▶ So far, considered electrons thermally excited across barrier
- ▶ Will there be a current at  $T = 0$ ?



## Fowler-Nordheim tunneling

- ▶ Consider very strong electric field  $\mathcal{E}$ ; neglect Schottky effect
- ▶ Minimum energy of electron in vacuum  $E_{\min}(z) \approx E_F + \Phi - e\mathcal{E}z$
- ▶ Electrons in metal with energy  $E < E_F$  have less than minimum energy for  $0 < z < \frac{E_F + \Phi - E}{e\mathcal{E}}$
- ▶ Tunneling probability, accounting for  $z$ -KE:

$$T(p_z) \approx \exp \frac{-2 \int dz \sqrt{2mE_{\min}(z) - p_z^2}}{\hbar} \approx \exp \frac{-4\sqrt{2m} \left( E_F + \Phi - \frac{p_z^2}{2m} \right)^{3/2}}{3e\hbar\mathcal{E}}$$

(based on the semi-classical WKB approximation for wavefunctions)

- ▶ Tunneling current:

$$j = \int_{p < p_F} \frac{d\vec{p}}{(2\pi\hbar)^3} \frac{ep_z}{m} T(p_z) \approx \frac{e^3}{16\pi^2\hbar\Phi} \mathcal{E}^2 \exp \frac{4\sqrt{2m}\phi^3}{3e\hbar\mathcal{E}}$$

- ▶ Identical dependence with  $\mathcal{E}$ , as thermionic emission had with  $T$  (even though one strictly classical, other quantum mechanical)