HW9

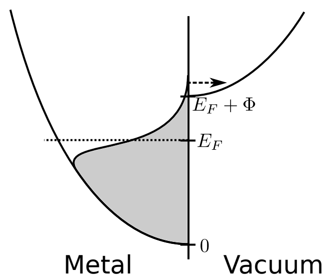
MTLE-6120: Spring 2018

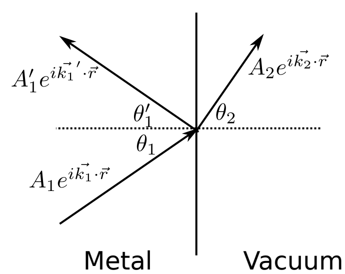
Due: Apr 16, 2018

1. **Electrons crossing an interface**

   An electron inside a free-electron metal is approaching its surface at incident angle θ_1 with respect to the normal, and is then either reflected or transmitted to vacuum with some probability. The work function of the metal is Φ, its Fermi level relative to the bottom of the band is E_F, and we choose to label the bottom of the metal's band as the reference energy $E = 0$. Assume that the electron mass is the free electron value m on both sides.

   (Use this problem to develop an intuitive conection between wave optics and quantum electron mechanics discussed earlier in the course. Waves be waves!)





   (a) What is the minimum electron energy E that can cross the interface at normal incidence ($\theta_1 = 0$)?

   (b) What are the dispersion relations $E(\vec{k})$ for electrons in the metal and in vacuum?

   (c) For an electron of energy E sufficient to cross the interface, what are the magnitudes $|\vec{k}_1|$, $|\vec{k}_1'|$ and $|\vec{k}_2|$ of the incident, reflected and transmitted electron wavevectors respectively?

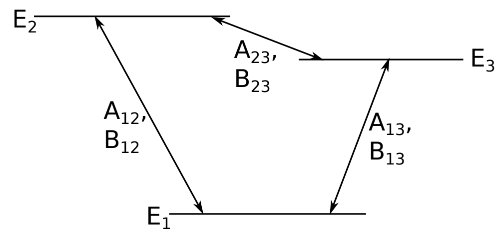
   (d) Using the phase-matching condition that the components of $\vec{k}$ in the plane of the interface must be equal for all three electron waves, derive Snell's law for the electron of energy E (i.e. what is the relation between θ_1, θ_1' and θ_2). Express your answer only in terms of E, E_F, Φ and any fundamental constants.

   (e) Write the matching conditions for the electron wavefunction across the interface and solve for the reflection and transmission amplitudes, $r \equiv A_1'/A_1$ and $t \equiv A_2/A_1$. Express the answer only in terms of $E, E_F, \Phi, \cos\theta_1$ and any fundamental constants. Hint: you only need to do this at one point, which you can set as $\vec{r} = 0$ for convenience.

   (f) What are the conditions for total internal reflection, and for zero reflection?

2. **Optical pumping and fluorescence**

   Cosnider the minimal three-level system necessary for fluorescence (and lasing) as shown below. A pump light of intensity I_{pump} is tuned to a frequency matching $E_2 - E_1$, and assume the intensities at other frequencies are small enough that stimulated emission is negligible for the other transitions. Assume A and B coefficients for each pair of states as shown.



(a) Write the differential equations governing the kinetics of N_1, N_2 and N_3: the populations (electrons / volume) for the three states.

(b) In steady state, find the ratio N_3/N_1 in order to determine the condition for population inversion ($N_3 > N_1$). Find and interpret the $I_{\text{pump}} \to \infty$ limit of this criterion.

(c) What is the net power density (rate of energy change per unit volume) absorbed from the pump light into the electrons? Assume that stimulated emission puts energy back into I_{pump} (best case scenario for efficiency), while the energy from spontaneous emission is lost. Just write the answer in terms of instantaneous N_1, N_2, N_3 (don't solve for the Ns).

(d) Similarly, what is the power density output from the $3 \to 1$ transition (fluorescence)? Again, just express in terms of N_1, N_2, N_3 as needed.

(e) What is the energy efficiency of the fluorescence process in steady state, and how does it depend on I_{pump}? (This time, solve for the Ns in terms of the A, B parameters and interpret!)