HW9

MTLE-6120: Spring 2023

Due: March 31, 2023

1. Kasap 8.17: Superconductivity and critical current density

2. Critical magnetic field of a superconductor

Consider a metal with some density of states per unit volume g(E) in its normal state, which becomes a BCS superconductor below a critical temperature T_c . In BCS theory, the superconducting gap at zero temperature is given by $\Delta = 1.76k_BT_c$.

- (a) If all the electrons with energy $E_F \Delta < E < E_F$ pair up with binding energy Δ per pair, what is the gain in energy density of the superconductor relative to the normal metal?
- (b) What is the energy density incurred in expelling a magnetic field B due to the Meissner effect?
- (c) Given that at the critical magnetic field B_c , it is no longer energetically favorable to expel the magnetic field, relate B_c to Δ (at T = 0).
- (d) Aluminum is face-centered cubic metal with a cubic lattice constant of 4.05 Å, which behaves almost perfectly like a free-electron metal with 3 free electrons per atom. What is its $g(E_F)$ in SI units $(J^{-1}m^{-3})$?
- (e) Given that aluminum becomes a BCS superconductor below $T_c = 1.2$ K, estimate its zerotemperature critical magnetic field B_c in SI units (Tesla).