

HW8 solution

MTLE-6120: Spring 2023

Due: March 24, 2023

1. Kasap 7.20: Capacitor design

- (a) For low voltage application, one can use the minimum thickness to minimize the volume. The capacitance is then $C = \epsilon_r \epsilon_0 A / d_{\min} = \epsilon_r \epsilon_0 V / d_{\min}^2$, so that the volume is $C d_{\min}^2 / (\epsilon_0 \epsilon_r)$. Substituting from table 7.13

	PET	TiO ₂	BaTiO ₃
Volume [m ³]	$(3.5 - 14) \times 10^{-9}$	1.26×10^{-8}	6.3×10^{-10}

Note the significant reduction in volume achievable with high dielectric materials, even with a higher minimum thickness. This makes BaTiO₃ produce the lowest volume.

- (b) For high voltage applications, the thickness may be limited by the breakdown field. We now require $V_{\max}/d < \mathcal{E}_{\text{br}}/2$ i.e. $d = 2V_{\max}/\mathcal{E}_{\text{br}}$

	PET	TiO ₂	BaTiO ₃
d [μm]	67	200	100
Volume [m ³]	1.6×10^{-5}	5×10^{-6}	6.3×10^{-8}

The high dielectric constant of BaTiO₃ still wins in minimizing the volume, and by an even larger margin because breakdown limits the thickness of PET by a larger factor than it does the remaining two.

- (c) The dissipated power is $P = CV^2\omega \tan \delta = 2\pi V^2\nu \tan \delta$. Substituting $C = 100$ nF and $\nu = 60$ Hz:

	PET	TiO ₂	BaTiO ₃
P [W]	0.047	0.0038	0.47

Note that in case the power dissipation is determined entirely by the loss tangent $\tan \delta$, since the other parameters are constant across materials for the same capacitance and operating conditions. The lowest dissipation is therefore for the lowest loss tangent material, TiO₂.

2. Kasap 8.6: Ferromagnetism and the exchange interaction

Note that there are effectively three parts to this question, as listed below for clarity:

- (a) What is the spin magnetic moment of the isolated Dy atom based on its electronic configuration?
The spin magnetic moment is due to $4f^{10}$, which given that there are seven f orbitals, corresponds to $\uparrow\downarrow, \uparrow\downarrow, \uparrow\downarrow, \uparrow, \uparrow, \uparrow, \uparrow$, and therefore four unpaired spins. The corresponding magnetic moment is $4 \times g\mu_B/2 \approx 4\mu_B$.
- (b) What is the magnetic moment per Dy atom in the solid based on the saturation magnetization? Compare with (a).

From the density and atomic mass, the number density of atoms is $(8.54 \text{ g/cm}^3) / (162.50 \text{ g/mol}) \times 6.022 \times 10^{23} / \text{mol} = 3.16 \times 10^{22} / \text{cm}^3$. The saturation magnetization per unit volume is $2.4 \times 10^6 \text{ A/m}$, which corresponds to $(2.4 \times 10^6 \text{ A/m}) / (3.16 \times 10^{22} / \text{cm}^3) = 7.6 \times 10^{-23} \text{ Am}^2$ per atom.

Since the Bohr magneton is $9.274 \times 10^{-24} \text{ Am}^2$, this magnetization corresponds to $8.2 \mu_B$ or 8.2 spins per atom.

Therefore the magnetization per atom in the solid is twice that in the isolated atom. This is perfectly reasonable because in metals, the number of unpaired spins depends on where the Fermi level is relative to the band density of states. This can differ considerably from the atomic picture.

- (c) Estimate the exchange interaction magnitude given the Curie temperature.

Assuming $E_{ex} \sim k_B T_c$ with $T_c \sim 85 \text{ K}$ yields $E_{ex} \sim 7.4 \times 10^{-3} \text{ eV}$.