HW8

MTLE-6120: Spring 2018

Due: Apr 9, 2018

1. **Kasap 8.6: Ferromagnetism and the exchange interaction**

   Note that there are effectively three parts to this question, as listed below for clarity:

   (a) What is the spin magnetic moment of the isolated Dy atom based on its electronic configuration?

   (b) What is the magnetic moment per Dy atom in the solid based on the saturation magnetization? Compare with (a).

   (c) Estimate the exchange interaction magnitude given the Curie temperature.

2. **Kasap 8.17: Superconductivity and critical current density**

3. **Critical magnetic field of a superconductor**

   Consider a metal with some density of states per unit volume $g(E)$ in its normal state, which becomes a BCS superconductor below a critical temperature T_c. In BCS theory, the superconducting gap at zero temperature is given by $\Delta = 1.76 k_B T_c$.

   (a) If all the electrons with energy $E_F - \Delta < E < E_F$ pair up with binding energy Δ per pair, what is the gain in energy density of the superconductor relative to the normal metal?

   (b) What is the energy density incurred in expelling a magnetic field B due to the Meissner effect?

   (c) Given that at the critical magnetic field B_c, it is no longer energetically favorable to expel the magnetic field, relate B_c to Δ (at $T = 0$).

   (d) Aluminum is face-centered cubic metal with a cubic lattice constant of 4.05 Å, which behaves almost perfectly like a free-electron metal with 3 free electrons per atom. What is its $g(E_F)$ in SI units ($J^{-1}m^{-3}$)?

   (e) Given that aluminum becomes a BCS superconductor below $T_c = 1.2$ K, estimate its zero-temperature critical magnetic field B_c in SI units (Tesla).