HW7 solution

MTLE-6120: Spring 2023

Due: March 17, 2023

1. **Kasap 5.11: Ionization at low temperatures in doped semiconductors**

   Note that there are effectively three questions in that problem, listed out below for clarity:

   (a) Show equation 5.85 for <u>n-doping</u>.

   Start from the full condition for charge-neutrality as derived in class

$$0 = -\underbrace{N_c e^{-\frac{E_g - E_F}{k_B T}}}_{n} + \underbrace{N_v e^{-\frac{E_F}{k_B T}}}_{p} + \frac{N_d}{1 + g_d e^{\frac{E_F - E_d}{k_B T}}} - \frac{N_a}{1 + g_a e^{\frac{E_a - E_F}{k_B T}}}$$

   For n-type doping, we set N_a to zero and neglect the hole contribution p, so that

$$n = N_c e^{-\frac{E_g - E_F}{k_B T}} = \frac{N_d}{1 + g_d e^{\frac{E_F - E_d}{k_B T}}}$$

   We need to eliminate E_F to get to 5.85 which is written in terms of n instead. Writing the first part of the equation as $e^{\frac{E_F}{k_B T}} = \frac{n}{N_c} e^{\frac{E_g}{k_B T}}$ and substituting into the last part:

$$n = \frac{N_d}{1 + \frac{g_d n}{N_c} e^{\frac{E_g - E_d}{k_B T}}}$$

   Defining $\Delta E = E_c - E_d = E_g - E_d$ (in our convention of $E_v = 0$), and rearranging we get

$$n + \frac{g_d n^2}{N_c} e^{\frac{\Delta E}{k_B T}} = N_d$$

   which upon dividing by the coefficient of n^2 is the same as 5.85:

$$n^2 + \frac{n N_c}{g_d \exp \frac{\Delta E}{k_B T}} - \frac{N_d N_c}{g_d \exp \frac{\Delta E}{k_B T}} = 0$$

   (b) Show that 5.85 reduces to 5.19 at low temperatures.

   Equation 5.85 is a quadratic equation in n, which has one positive root:

$$n = \frac{1}{2} \left[-\frac{N_c}{g_d \exp \frac{\Delta E}{k_B T}} + \sqrt{\left(\frac{N_c}{g_d \exp \frac{\Delta E}{k_B T}} \right)^2 + 4 \frac{N_d N_c}{g_d \exp \frac{\Delta E}{k_B T}}} \right]$$

At low temperatures, the second term inside the $\sqrt{}$ dominates because $\exp \frac{\Delta E}{k_B T}$ is small and the first term has that factor twice. Then the overall $\sqrt{}$ will be $\sim \exp \frac{\Delta E}{2k_B T}$, which is much larger than the $\sim \exp \frac{\Delta E}{k_B T}$ outside. Therefore:

$$n \approx \frac{1}{2}\sqrt{4\frac{N_d N_c}{g_d \exp \frac{\Delta E}{k_B T}}} = \sqrt{\frac{N_d N_c}{g_d}} \exp \frac{-\Delta E}{2k_B T}$$

which is the same as equation 5.19 since $g_d = 2$.

(c) Estimate 90% ionization temperature for Ga <u>p-doping</u> in Si.

We can get the equation analogous to 5.85 for p-type doping by switching $n \leftrightarrow p$, $N_c \leftrightarrow N_v$ and $g_d \leftrightarrow g_a$. Now $\Delta E = E_a - E_v$, still the distance of the dopant (acceptor) level from the corresponding band. Therfore we have:

$$p^2 + \frac{(p - N_a)N_v}{g_a \exp \frac{\Delta E}{k_B T}} = 0$$

At 90% ionization, $p = 0.9N_a$ (assuming $N_a \gg n_i$) so that

$$(0.9N_a)^2 + \frac{(0.9N_a - N_a)N_v}{g_a \exp \frac{\Delta E}{k_B T}} = 0$$

which we can rearrange as a solution for T as:

$$T = \frac{\Delta E}{k_B \ln \frac{N_v}{8.1 N_a g_a}}$$

Substituting $\Delta E = 0.065$ eV, $N_v = 1.2 \times 10^{19}$ cm^{-3}, $N_a = 10^{15}$ cm^{-3} and $g_a = 4$ yields 128 K $=$ -145 C.

2. **Kasap 5.18: Hall effect in semiconductors**

Hint: maybe do part (b) first to get some intuition.

(a) The Hall coefficient as a function of n alone is given by

$$R_H = \frac{p - nb^2}{e(p + nb)^2} = \frac{n_i^2/n - nb^2}{e(n_i^2/n + nb)^2} = \frac{n_i^2 n - n^3 b^2}{e(n_i^2 + n^2 b)^2} = \frac{x(1 - x^2 b^2)}{n_i e(1 + x^2 b)^2}$$

where $x \equiv n/n_i$. To find the extrema:

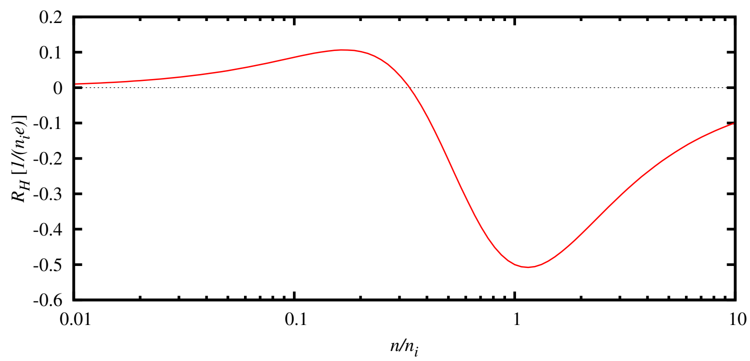
$$\begin{aligned}
0 &= \frac{\mathrm{d}R_H}{\mathrm{d}x} \\
&= \frac{(1 - 3x^2b^2)(1 + x^2 b)^2 - (x - x^3 b^2)2(1 + x^2 b)2xb}{n_i e(1 + x^2 b)^4} \\
&= \frac{(1 + x^2 b)}{n_i e(1 + x^2 b)^3}\left[(1 - 3x^2 b^2)(1 + x^2 b) - (x - x^3 b^2)4xb\right] \\
\Rightarrow \qquad 0 &= (1 - 3x^2 b^2)(1 + x^2 b) - (x - x^3 b^2)4xb \\
&= x^4 b^3 - 3x^2 b(b + 1) + 1 \\
\Rightarrow \qquad \frac{n}{n_i} = x &= \sqrt{\frac{3b(b+1) \pm \sqrt{9b^2(b+1)^2 - 4b^3}}{2b^3}} \\
&= \sqrt{\frac{3(b+1) \pm \sqrt{9(b+1)^2 - 4b}}{2b^2}}
\end{aligned}$$

Note that the $-$ root corresponds to $n/n_i > 1/b$, which makes R_H negative (electron-dominated) while the $+$ root corresponds to $n/n_i < 1/b$, which makes R_H positive (hole-dominated).

Basically, R_H is zero when $n = bn_i$. It increases in magnitude when n either increases or decreases, reaching maximum magnitudes for the above solutions. Beyond that it decreases in magnitude $\propto 1/n$ for large n and $\propto n$ (i.e. $1/p$) for small n (as we show below in (c)).

For silicon, $b = \mu_e/\mu_h \approx 3$, so that $n/n_i = \sqrt{(12 \pm \sqrt{132})/18} \approx 0.17$ and 1.14.

(b) Using the final form of R_H above in terms of $x = n/n_i$, we can plot:



which shows the trend explained in (a) with the maximum magnitudes at the indentified locations.

(c) When $n \gg n_i$, p is negligible compared to n, so that the first form of R_H above reduces to $-nb^2/(e(nb)^2) = -1/(ne)$.

Similarly, when $n \ll n_i$, n is negligible compared to p, so that the first form of R_H above reduces to $p/(e(p)^2) = +1/(pe)$.

3. **Kasap 5.20: Compound semiconductor devices**

(a) The band gap of 0.67 eV corresponds to a wavelength $\lambda \approx 1240$ eV-nm/(0.67 eV) = 1850 nm, which is in the infrared portion of the spectrum.

(b) The intrinsic carrier density of GaSb is given by

$$n_i = \sqrt{N_c N_v} \exp \frac{E_g}{2k_B T} \approx 8.8 \times 10^{13}\ \mathrm{cm}^{-3}$$

The intrinsic conductivity is $\sigma_i = n_i e(\mu_e + \mu_h) = 8.8 \times 10^{13} \cdot 1.602 \times 10^{-19} \cdot (5000 + 1000)$ A/(V cm) $\approx 8.5\ (\Omega\mathrm{m})^{-1}$.

In comparison, for Ge, $n_i \approx 2.3 \times 10^{13}$ cm^{-3} and e and h mobilities are 3900 and 1900 cm^2/(V s), which yields $\sigma_i = 2.3 \times 10^{13} \cdot 1.602 \times 10^{-19} \cdot (3900 + 1900)$ A/(V cm) $\approx 2.1\ (\Omega\mathrm{m})^{-1}$.

The bandgaps are very similar, so the intrinsic conductivities are of the same order of magnitude. However, the band-edge density of states are higher in GaSb, yielding a higher conductivity.

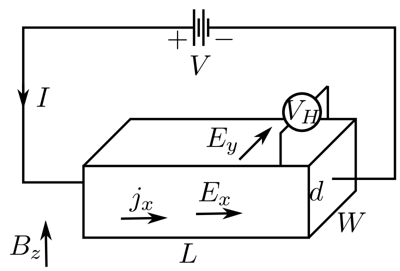
(c) Excess Sb leads to an n-type semiconductor, so to get conductivity $\sigma = 100\ (\Omega\mathrm{cm})^{-1}$, we need extra Sb with concentration $N_d = \sigma/(\mu_e e) \approx 1.25 \times 10^{17}$ cm^{-3}.

The atomic mass of one GaSb unit is (69.7 + 121.8) = 191.5 g/mol. Thus the number density of Ga and Sb atoms is (5.4 g/cm^3)/(191.5 g/mol) $\cdot\, 6.022 \times 10^{23}$/mol $\approx 1.7 \times 10^{22}$ cm^{-3}. Therefore, the required doping level is $\delta = 1.25 \times 10^{17}/1.7 \times 10^{22} \approx 7.4 \times 10^{-6}$ (i.e. 7.4 ppm).

4. **Semiconductor hall probe**

We want to design a sensitive microscopic magnetic field sensor using the Hall effect in an n-type semiconductor. Assume that we work with a doping level $N_d \gg n_i$ (so that hole contributions are negligible), and in the rectangular geometry shown below.

Further, assume that we need 1 μm spatial resolution, so that we set $L = W = 1$ μm. We also need to operate using a voltage source with $V = 1$ V. The parameters we need to design are the thickness d and the doping level N_d. Assume for simplicity that the electron mobility is $\mu_e = 1000$ cm^2/(Vs), independent of N_d.



(a) Express the sensitivity of measured Hall voltage to magnetic field, $\mathrm{d}V_H/\mathrm{d}B$ in terms of the device geometry, voltage and mobility. Do not substitute any values yet.

The Hall coefficient magnitude is $1/(N_d e)$, which relates the Hall field V_H/W to the product of current density j and the magnetic field B. Therefore:

$$\frac{V_H}{W} = \frac{1}{N_d e} \cdot jB$$

which yields

$$\frac{\mathrm{d}V_H}{\mathrm{d}B} = \frac{jW}{N_d e}.$$

The current density $j = \sigma E$, where the electric field $E = V/L$ and the conductivity $\sigma = N_d \mu_e e$, which substituting above yields

$$\frac{\mathrm{d}V_H}{\mathrm{d}B} = \frac{W}{N_d e} \cdot N_d \mu_e e \cdot \frac{V}{L} = \mu_e V \frac{W}{L}.$$

(b) How does the sensitivity depend on the undetermined design parameters N_d and d? Which material properties most affect this sensitivity?

The sensitivity does not depend on our unknown parameters at all! The only geometric factor affecting it is W/L, which we have already chosen to be 1. Besides that, it is only affected by the electron mobility and the applied voltage (directly proportional to both).

(c) What is the condition on $N_d \cdot d$ such that the power dissipated in the semiconductor is less than 1 mW? Substitute values and express result in cm^{-2} units.

The power dissipation in the semiconductor is:

$$\frac{V^2}{R} = \frac{V^2 \sigma W d}{L} = \frac{V^2 N_d \mu_e e W d}{L} \leq P_{\max}$$

Therefor the condition we seek can be expressed as:

$$N_d d \leq \frac{P_{\max} L}{V^2 \mu_e e W} \approx 6.25 \times 10^{12}\ \mathrm{cm}^{-2}$$

(d) These Hall probes are intended to be used in an array to map magnetic fields, and should ideally all exhibit the same electrical characteristics. One issue is that if there are N dopants on average per device, statictical fluctuations produce variations on the order of $\sqrt{N}$. What is the condition on $N_d \cdot d$ such that the variability of electrical characteristics is less than 1%? Substitute values and express result in cm^{-2} units.

The number of dopants in one device is $N = N_d d W L$. For statictical fluctuations to be less than 1%, we need $\sqrt{N}/N \leq 0.01 \Rightarrow N \geq 10^4$. Therefore, $N_d d \geq 10^4/(WL) = 10^{12}$ cm^{-2}.

(e) Set $N_d \cdot d$ to be the geometric mean of the minimum and maximum values determined in the previous two parts. Calculate the sensitivity $\mathrm{d}V_H/\mathrm{d}B$ of the Hall voltage to magnetic field in V/T units.

The sensitivity of the Hall probe determined above is $\mu_e V \frac{W}{L} = \mu_e V$ for the $L = W$ choice, independent of N_d and d. Therefore the sensitivity is $1000\ \mathrm{cm}^2/(\mathrm{Vs}) \cdot 1\ \mathrm{V} = 0.1\ \mathrm{m}^2/\mathrm{s} = 0.1\ \mathrm{V/T}$.