HW7 solution

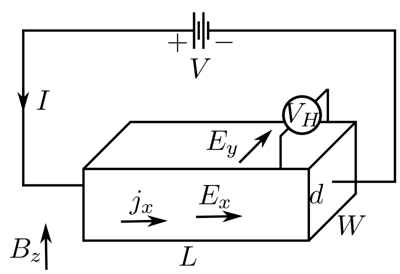
MTLE-6120: Spring 2018

Due: Apr 2, 2018

1. **Semiconductor hall probe**

   We want to design a sensitive microscopic magnetic field sensor using the Hall effect in an n-type semiconductor. Assume that we work with a doping level $N_d \gg n_i$ (so that hole contributions are negligible), and in the rectangular geometry shown below.

   Further, assume that we need 1 μm spatial resolution, so that we set $L = W = 1$ μm. We also need to operate using a voltage source with $V = 1$ V. The parameters we need to design are the thickness d and the doping level N_d. Assume for simplicity that the electron mobility is $\mu_e = 1000$ cm^2/(Vs), independent of N_d.



   (a) Express the sensitivity of measured Hall voltage to magnetic field, $\mathrm{d}V_H/\mathrm{d}B$ in terms of the device geometry, voltage and mobility. Do not substitute any values yet.

   The Hall coefficient magnitude is $1/(N_d e)$, which relates the Hall field V_H/W to the product of current density j and the magnetic field B. Therefore:

$$\frac{V_H}{W} = \frac{1}{N_d e} \cdot jB$$

   which yields

$$\frac{\mathrm{d}V_H}{\mathrm{d}B} = \frac{jW}{N_d e}.$$

   The current density $j = \sigma E$, where the electric field $E = V/L$ and the conductivity $\sigma = N_d \mu_e e$, which substituting above yields

$$\frac{\mathrm{d}V_H}{\mathrm{d}B} = \frac{W}{N_d e} \cdot N_d \mu_e e \cdot \frac{V}{L} = \mu_e V \frac{W}{L}.$$

   (b) How does the sensitivity depend on the undetermined design parameters N_d and d? Which material properties most affect this sensitivity?

   The sensitivity does not depend on our unknown parameters at all! The only geometric factor affecting it is W/L, which we have already chosen to be 1. Besides that, it is only affected by the electron mobility and the applied voltage (directly proportional to both).

(c) What is the condition on $N_d \cdot d$ such that the power dissipated in the semiconductor is less than 1 mW? Substitute values and express result in cm^{-2} units.

The power dissipation in the semiconductor is:

$$\frac{V^2}{R} = \frac{V^2 \sigma W d}{L} = \frac{V^2 N_d \mu_e e W d}{L} \leq P_{\max}$$

Therefor the condition we seek can be expressed as:

$$N_d d \leq \frac{P_{\max} L}{V^2 \mu_e e W} \approx 6.25 \times 10^{12}\ \mathrm{cm}^{-2}$$

(d) These Hall probes are intended to be used in an array to map magnetic fields, and should ideally all exhibit the same electrical characteristics. One issue is that if there are N dopants on average per device, statictical fluctuations produce variations on the order of $\sqrt{N}$. What is the condition on $N_d \cdot d$ such that the variability of electrical characteristics is less than 1%? Substitute values and express result in cm^{-2} units.

The number of dopants in one device is $N = N_d d W L$. For statictical fluctuations to be less than 1%, we need $\sqrt{N}/N \leq 0.01 \Rightarrow N \geq 10^4$. Therefore, $N_d d \geq 10^4/(WL) = 10^{12}$ cm^{-2}.

(e) Set $N_d \cdot d$ to be the geometric mean of the minimum and maximum values determined in the previous two parts. Calculate the sensitivity $\mathrm{d}V_H/\mathrm{d}B$ of the Hall voltage to magnetic field in V/T units.

The sensitivity of the Hall probe determined above is $\mu_e V \frac{W}{L} = \mu_e V$ for the $L = W$ choice, independent of N_d and d. Therefore the sensitivity is $1000\ \mathrm{cm}^2/(\mathrm{Vs}) \cdot 1\ \mathrm{V} = 0.1\ \mathrm{m}^2/\mathrm{s} = 0.1$ V/T.

2. **Kasap 7.20: Capacitor design**

(a) For low voltage application, one can use the minimum thickness to minimize the volume. The capacitance is then $C = \epsilon_r \epsilon_0 A/d_{\min} = \epsilon_r \epsilon_0 V/d_{\min}^2$, so that the volume is $C d_{\min}^2/(\epsilon_0 \epsilon_r)$. Substituting from table 7.13

	PET	TiO_2	$BaTiO_3$
Volume [m^3]	$(3.5 - 14) \times 10^{-9}$	1.26×10^{-8}	6.3×10^{-10}

Note the significant reduction in volume achievable with high dielectric materials, even with a higher minimum thickness. This makes $BaTiO_3$ produce the lowest volume.

(b) For high voltage applications, the thickness may be limited by the breakdown field. We now require $V_{\max}/d < \mathcal{E}_{\mathrm{br}}/2$ i.e. $d = 2V_{\max}/\mathcal{E}_{\mathrm{br}}$

	PET	TiO_2	$BaTiO_3$
d [μm]	67	200	100
Volume [m^3]	1.6×10^{-5}	5×10^{-6}	6.3×10^{-8}

The high dielectric constant of $BaTiO_3$ still wins in minimizing the volume, and by an even larger margin because breakdown limits the thickness of PET by a larger factor than it does the remaining two.

(c) The dissipated power is $P = CV^2 \omega \tan\delta = 2\pi V^2 \nu \tan\delta$. Substituting $C = 100$ nF and $\nu = 60$ Hz:

	PET	TiO_2	$BaTiO_3$
P [W]	0.047	0.0038	0.47

Note that in case the power dissipation is determined entirely by the loss tangent $\tan\delta$, since the other parameters are constant across materials for the same capacitance and operating conditions. The lowest dissipation is therefore for the lowest loss tangent material, TiO_2.