

HW6 solution

MTLE-6120: Spring 2018

Due: Mar 12, 2018

1. Fermi theory properties of a metal in 2D

Consider a simple metal in 2D containing n_{ion} ions per unit area. For all questions asking for a ‘dependence’ below, only predict the power law eg. explain why something is T^3 (say), without worrying about the prefactor.

- (a) What is the temperature dependence of electronic heat capacity for $T \ll T_F$?
The Fermi theory arguments for $T \ll T_F$ work regardless of the electronic DOS details. Nothing changes in going from 2D to 3D, and $C_V \propto T$ for electrons.
- (b) What is the frequency dependence of the phonon density of states $g(\omega)$ in the Debye model?
As derived in class, the density of states of phonons with linear dispersion is $\propto \omega^{d-1}$ i.e. $\propto \omega$. This is cutoff at the Debye frequency ω_D , so the overall frequency dependence is $g(\omega) \propto \omega \Theta(\omega_D - \omega)$.
- (c) What is the temperature dependence of the lattice heat capacity for $T \ll T_D$?
Compared to the derivation for the 3D case in class, all that changes is you have one less power of ω in the phonon density of states and hence the internal energy integral. In the $T \ll T_D$ limit, this gives one fewer power of T , which yields $C_V \propto T^2$.
- (d) What is the lattice heat capacity per unit area for $T \gg T_D$? (Give absolute expression.)
Equipartition theorem applies, so $C_V = 2n_{\text{ion}}k_B$. Note 2 instead of 3, because you have 2 degrees of freedom per ion now, instead of 3.
- (e) What is the temperature dependence of electronic resistivity for $T \ll T_D$?
The phonon density of states has one fewer power of ω , so the Fermi Golden rule integral has one fewer power too. In the $T \ll T_D$ limit, this gives one fewer power of T than the 3D case, i.e. $\rho \propto T^4$.
- (f) What is the temperature dependence of electronic resistivity for $T \gg T_D$?
For $T \gg T_D$, equipartition applies to phonons, so phonon DOS is irrelevant. Therefore the temperature dependence is same as 3D i.e. $\rho \propto T$.

2. Kasap 5.11: Ionization at low temperatures in doped semiconductors

Note that there are effectively three questions in that problem, listed out below for clarity:

- (a) Show equation 5.85 for n -doping.
Start from the full condition for charge-neutrality as derived in class

$$0 = -\underbrace{N_c e^{-\frac{E_g - E_F}{k_B T}}}_n + \underbrace{N_v e^{-\frac{E_F}{k_B T}}}_p + \frac{N_d}{1 + g_d e^{\frac{E_F - E_d}{k_B T}}} - \frac{N_a}{1 + g_a e^{\frac{E_a - E_F}{k_B T}}}$$

For n -type doping, we set N_a to zero and neglect the hole contribution p , so that

$$n = N_c e^{-\frac{E_g - E_F}{k_B T}} = \frac{N_d}{1 + g_d e^{\frac{E_F - E_d}{k_B T}}}$$

We need to eliminate E_F to get to 5.85 which is written in terms of n instead. Writing the first part of the equation as $e^{\frac{E_g}{k_B T}} = \frac{n}{N_c} e^{\frac{E_g}{k_B T}}$ and substituting into the last part:

$$n = \frac{N_d}{1 + \frac{g_d n}{N_c} e^{\frac{E_g - E_d}{k_B T}}}$$

Defining $\Delta E = E_c - E_d = E_g - E_d$ (in our convention of $E_v = 0$), and rearranging we get

$$n + \frac{g_d n^2}{N_c} e^{\frac{\Delta E}{k_B T}} = N_d$$

which upon dividing by the coefficient of n^2 is the same as 5.85:

$$n^2 + \frac{n N_c}{g_d \exp \frac{\Delta E}{k_B T}} - \frac{N_d N_c}{g_d \exp \frac{\Delta E}{k_B T}} = 0$$

(b) Show that 5.85 reduces to 5.19 at low temperatures.

Equation 5.85 is a quadratic equation in n , which has one positive root:

$$n = \frac{1}{2} \left[-\frac{N_c}{g_d \exp \frac{\Delta E}{k_B T}} + \sqrt{\left(\frac{N_c}{g_d \exp \frac{\Delta E}{k_B T}} \right)^2 + 4 \frac{N_d N_c}{g_d \exp \frac{\Delta E}{k_B T}}} \right]$$

At low temperatures, the second term inside the $\sqrt{\quad}$ dominates because $\exp \frac{\Delta E}{k_B T}$ is small and the first term has that factor twice. Then the overall $\sqrt{\quad}$ will be $\sim \exp \frac{\Delta E}{2k_B T}$, which is much larger than the $\sim \exp \frac{\Delta E}{k_B T}$ outside. Therefore:

$$n \approx \frac{1}{2} \sqrt{4 \frac{N_d N_c}{g_d \exp \frac{\Delta E}{k_B T}}} = \sqrt{\frac{N_d N_c}{g_d}} \exp \frac{-\Delta E}{2k_B T}$$

which is the same as equation 5.19 since $g_d = 2$.

(c) Estimate 90% ionization temperature for Ga p -doping in Si.

We can get the equation analogous to 5.85 for p -type doping by switching $n \leftrightarrow p$, $N_c \leftrightarrow N_v$ and $g_d \leftrightarrow g_a$. Now $\Delta E = E_a - E_v$, still the distance of the dopant (acceptor) level from the corresponding band. Therefore we have:

$$p^2 + \frac{(p - N_a) N_v}{g_a \exp \frac{\Delta E}{k_B T}} = 0$$

At 90% ionization, $p = 0.9N_a$ (assuming $N_a \gg n_i$) so that

$$(0.9N_a)^2 + \frac{(0.9N_a - N_a) N_v}{g_a \exp \frac{\Delta E}{k_B T}} = 0$$

which we can rearrange as a solution for T as:

$$T = \frac{\Delta E}{k_B \ln \frac{N_v}{8.1N_a g_a}}$$

Substituting $\Delta E = 0.065$ eV, $N_v = 1.2 \times 10^{19}$ cm⁻³, $N_a = 10^{15}$ cm⁻³ and $g_a = 4$ yields 128 K = -145 C.

3. Kasap 5.18: Hall effect in semiconductors

Hint: maybe do part (b) first to get some intuition.

(a) The Hall coefficient as a function of n alone is given by

$$R_H = \frac{p - nb^2}{e(p + nb)^2} = \frac{n_i^2/n - nb^2}{e(n_i^2/n + nb)^2} = \frac{n_i^2 n - n^3 b^2}{e(n_i^2 + n^2 b)^2} = \frac{x(1 - x^2 b^2)}{n_i e(1 + x^2 b)^2}$$

where $x \equiv n/n_i$. To find the extrema:

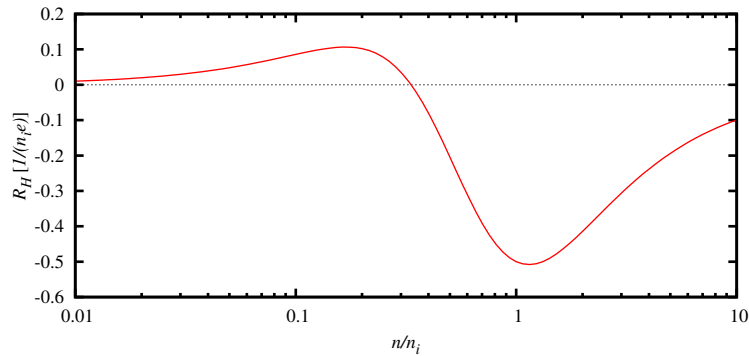
$$\begin{aligned} 0 &= \frac{dR_H}{dx} \\ &= \frac{(1 - 3x^2 b^2)(1 + x^2 b)^2 - (x - x^3 b^2)2(1 + x^2 b)2xb}{n_i e(1 + x^2 b)^4} \\ &= \frac{(1 + x^2 b)}{n_i e(1 + x^2 b)^3} [(1 - 3x^2 b^2)(1 + x^2 b) - (x - x^3 b^2)4xb] \\ \Rightarrow 0 &= (1 - 3x^2 b^2)(1 + x^2 b) - (x - x^3 b^2)4xb \\ &= x^4 b^3 - 3x^2 b(b + 1) + 1 \\ \Rightarrow \frac{n}{n_i} = x &= \sqrt{\frac{3b(b + 1) \pm \sqrt{9b^2(b + 1)^2 - 4b^3}}{2b^3}} \\ &= \sqrt{\frac{3(b + 1) \pm \sqrt{9(b + 1)^2 - 4b}}{2b^2}} \end{aligned}$$

Note that the $-$ root corresponds to $n/n_i > 1/b$, which makes R_H negative (electron-dominated) while the $+$ root corresponds to $n/n_i < 1/b$, which makes R_H positive (hole-dominated).

Basically, R_H is zero when $n = bn_i$. It increases in magnitude when n either increases or decreases, reaching maximum magnitudes for the above solutions. Beyond that it decreases in magnitude $\propto 1/n$ for large n and $\propto n$ (i.e. $1/p$) for small n (as we show below in (c)).

For silicon, $b = \mu_e/\mu_h \approx 3$, so that $n/n_i = \sqrt{(12 \pm \sqrt{132})/18} \approx 0.17$ and 1.14 .

(b) Using the final form of R_H above in terms of $x = n/n_i$, we can plot:



which shows the trend explained in (a) with the maximum magnitudes at the identified locations.

(c) When $n \gg n_i$, p is negligible compared to n , so that the first form of R_H above reduces to $-nb^2/(e(nb)^2) = -1/(ne)$.

Similarly, when $n \ll n_i$, n is negligible compared to p , so that the first form of R_H above reduces to $p/(e(p)^2) = +1/(pe)$.

4. Kasap 5.20: Compound semiconductor devices

- (a) The band gap of 0.67 eV corresponds to a wavelength $\lambda \approx 1240 \text{ eV}\cdot\text{nm}/(0.67 \text{ eV}) = 1850 \text{ nm}$, which is in the infrared portion of the spectrum.
- (b) The intrinsic carrier density of GaSb is given by

$$n_i = \sqrt{N_c N_v} \exp \frac{E_g}{2k_B T} \approx 8.8 \times 10^{13} \text{ cm}^{-3}$$

The intrinsic conductivity is $\sigma_i = n_i e (\mu_e + \mu_h) = 8.8 \times 10^{13} \cdot 1.602 \times 10^{-19} \cdot (5000 + 1000) \text{ A}/(\text{V cm}) \approx 8.5 (\Omega\text{m})^{-1}$.

In comparison, for Ge, $n_i \approx 2.3 \times 10^{13} \text{ cm}^{-3}$ and e and h mobilities are 3900 and 1900 $\text{cm}^2/(\text{V s})$, which yields $\sigma_i = 2.3 \times 10^{13} \cdot 1.602 \times 10^{-19} \cdot (3900 + 1900) \text{ A}/(\text{V cm}) \approx 2.1 (\Omega\text{m})^{-1}$.

The bandgaps are very similar, so the intrinsic conductivities are of the same order of magnitude. However, the band-edge density of states are higher in GaSb, yielding a higher conductivity.

- (c) Excess Sb leads to an *n*-type semiconductor, so to get conductivity $\sigma = 100 (\Omega\text{cm})^{-1}$, we need extra Sb with concentration $N_d = \sigma/(\mu_e e) \approx 1.25 \times 10^{17} \text{ cm}^{-3}$.

The atomic mass of one GaSb unit is $(69.7 + 121.8) = 191.5 \text{ g/mol}$. Thus the number density of Ga and Sb atoms is $(5.4 \text{ g/cm}^3)/(191.5 \text{ g/mol}) \cdot 6.022 \times 10^{23}/\text{mol} \approx 1.7 \times 10^{22} \text{ cm}^{-3}$. Therefore, the required doping level is $\delta = 1.25 \times 10^{17}/1.7 \times 10^{22} \approx 7.4 \times 10^{-6}$ (i.e. 7.4 ppm).