

HW3

MTLE-6120: Spring 2019

Due: Feb 11, 2019

1. **Kasap 3.20: X-Rays and the Moseley relation.**

Instead of frequency ν , perform the analysis using photon energy $\hbar\omega = h\nu$ in eV instead.

2. **Kasap 3.27: Hund's rule.**

3. **Photoemission from a δ -atom**

The δ -atom with potential $-V_0\delta(x)$ has a single bound state $e^{-\kappa|x|}\sqrt{\kappa}$ with $\kappa = mV_0/\hbar^2 \equiv q/2$ (as shown in class), and free states at any energy $E > 0$ (previous HW). Now apply a time-varying electric field $\mathcal{E}e^{-i\omega t}$ in the x direction (from an EM wave, for example).

- (a) For the atom in its ground state, what is the minimum ω that can free the electron?
- (b) For the δ -atom free states solved in the previous HW, let us solve the normalization problem by assuming that the entire system is inside an infinite potential well of length L (centered at $x = 0$). What are the normalized wavefunctions and the condition(s) satisfied by the allowed k ? (Hint: there are two classes of solutions: odd and even about $x = 0$. Also, you may not be able to solve for k explicitly in some cases; only write the condition satisfied by k in those cases.)
- (c) What are the matrix elements $\langle\psi_f|H'|\psi_i\rangle$ of the perturbation Hamiltonian $H' = e\mathcal{E}x$ between the bound state (initial) and the free states (final) determined above? Use $L \gg 1/k, 1/\kappa$ to simplify the integrals. (We are going to take $L \rightarrow \infty$ below.) Are there any selection rules?
- (d) Using Fermi's Golden rule, calculate the rate of photo-excitation out of the bound state. This requires summing over the final states which have separation in k that is proportional to $1/L$. Take the limit $L \rightarrow \infty$ so that this becomes an integral. Describe the frequency dependence of the photoemission rate.