HW2

MTLE-6120: Spring 2019

Due: Feb 4, 2019

1. **Counting waves**

   We showed in class that for waves trapped in a 3D box, the number of wave vectors $\vec{k}$ with magnitude between k and $k + \mathrm{d}k$ is $k^2 \mathrm{d}k/(2\pi^2)$ per unit volume, independent of box size (not counting the factor of 2 for polarizations here). What are the corresponding numbers for waves in 2D and 1D boxes? Optional: derive the general result for d dimensions.

2. **Photoelectric effect and quantum efficiency: Kasap 3.8**

3. **Scattering by a δ-atom**

   (a) Consider an electron with energy $E = \frac{\hbar^2 k^2}{2m} > 0$ incident from the left ($-\infty$) towards a δ-atom given by the potential $-V_0\delta(x)$, with strength parametrized using the parameter $q = 2mV_0/\hbar^2$. Find the wavefunction $\psi(x)$ (expressed in k and q, rather than E and V_0 for convenience). Can you normalize it?

   (b) What are the probabilities of the electron getting reflected and transmitted? Describe and explain their qualitative dependence on the energy of the electron. What would the corresponding probabilities be in the classical case?

   (c) Consider the opposite potential $+V_0\delta(x)$. What is the electron wavefunction? What are the probabilities of the electron getting reflected and transmitted? What would the corresponding probabilities be in the classical case?

   (d) Instead of a free electron with definite momentum k (and energy $E = \frac{\hbar^2 k^2}{2m}$) as considered above, consider a Gaussian wavepacket with coefficients

   $$c_k = \frac{1}{\sqrt{\sigma_k \sqrt{2\pi}}} \exp \frac{-(k - k_0)^2}{2\sigma_k^2},$$

   assuming that the spread in momentum $\sigma_k \ll k_0$ and q. Where is the electron as a function of time?