HW11

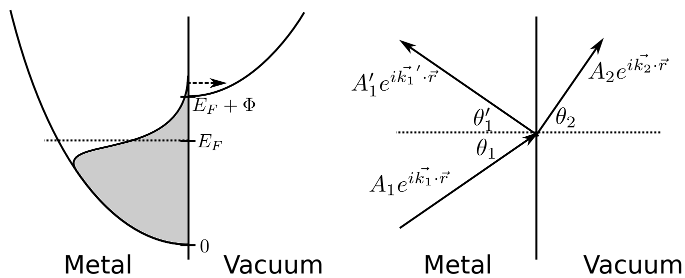
MTLE-6120: Spring 2023

Due: April 14, 2023

1. **Electrons crossing an interface**

   An electron inside a free-electron metal is approaching its surface at incident angle θ_1 with respect to the normal, and is then either reflected or transmitted to vacuum with some probability. The work function of the metal is Φ, its Fermi level relative to the bottom of the band is E_F, and we choose to label the bottom of the metal's band as the reference energy $E = 0$. Assume that the electron mass is the free electron value m on both sides.

   (Use this problem to develop an intuitive conection between wave optics and quantum electron mechanics discussed earlier in the course. Waves be waves!)



   (a) What is the minimum electron energy E that can cross the interface at normal incidence ($\theta_1 = 0$)?

   (b) What are the dispersion relations $E(\vec{k})$ for electrons in the metal and in vacuum?

   (c) For an electron of energy E sufficient to cross the interface, what are the magnitudes $|\vec{k}_1|$, $|\vec{k}_1'|$ and $|\vec{k}_2|$ of the incident, reflected and transmitted electron wavevectors respectively?

   (d) Using the phase-matching condition that the components of $\vec{k}$ in the plane of the interface must be equal for all three electron waves, derive Snell's law for the electron of energy E (i.e. what is the relation between θ_1, θ_1' and θ_2). Express your answer only in terms of E, E_F, Φ and any fundamental constants.

   (e) Write the matching conditions for the electron wavefunction across the interface and solve for the reflection and transmission amplitudes, $r \equiv A_1'/A_1$ and $t \equiv A_2/A_1$. Express the answer only in terms of $E, E_F, \Phi, \cos\theta_1$ and any fundamental constants. Hint: you only need to do this at one point, which you can set as $\vec{r} = 0$ for convenience.

   (f) What are the conditions for total internal reflection, and for zero reflection?

2. **Kasap 5.29: Seebeck coefficient and thermal drift in semiconductor devices**

3. **Kasap 6.2: The Si *pn* junction** (estimating recombination and diffusion currents)

   In answering, 'what is your conclusion', include your expectation for the diode ideality factor η.

   Hint: you will need the diffusion constants for the carriers. Assume $D_e = 34.9$ cm^2/s and $D_h = 11.6$ cm^2/s. (Diffusion constants are related to mobilities by the Einstein relation $D_{e/h} = (k_B T/e)\mu_{e/h}$) Besides that, the only other Si-specific properties you should need are $n_i = 10^{10}$ cm^{-3} and $\epsilon_r = 11.7$.

4. **Kasap 6.15: Ultimate limits to device performance** (of an n-channel FET)

   Note: for part (c), assume and justify reasonable values for the barriers. Consider tunneling to become important when tunneling probabilities $\sim 10^{-6}$.

   Also, there might be a typo in the example number referenced by some editions of the book. (In mine, it says 3.10, but it should be 3.12.)