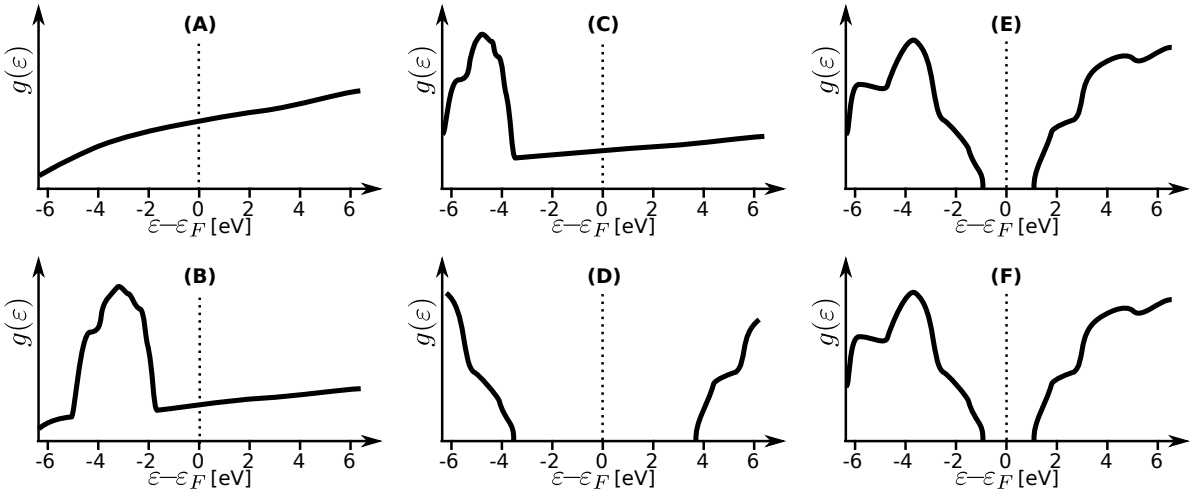
# HW10

MTLE-6120: Spring 2023

Due: April 7, 2023

1. **Optical property comparison**

   The density of states of six single-crystalline materials are sketched below. (Note that (E) and (F) are intentionally visually indistinguishable.)

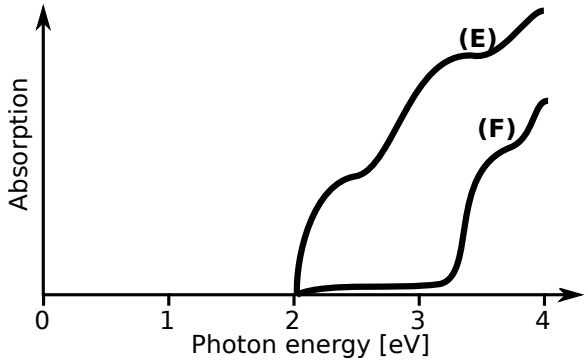



   (a) Identify each material as metallic, semiconducting or insulating.

   (b) Describe the visual appearance of crystals of each material.

   (c) What experiment would you perform to distinguish (A) and (C), and why? (Mention the results expected for each.)

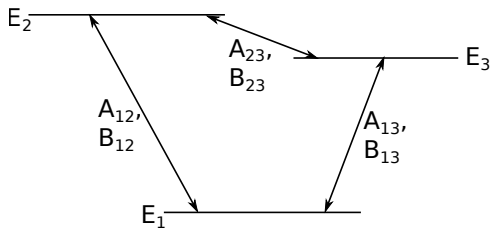
   (d) Despite nearly identical density of states, the absorption spectra of (E) and (F) happen to differ as shown below. Explain how this is possible, and what impact will it have on the appearance of these materials.




2. **Optical pumping and fluorescence**

   Consider the minimal three-level system necessary for fluorescence (and lasing) as shown below. A pump light of intensity $I_{\text{pump}}$ is tuned to a frequency matching $E_2 - E_1$, and assume the intensities

at other frequencies are small enough that stimulated emission is negligible for the other transitions. Assume $A$ and $B$ coefficients for each pair of states as shown.




(a) Write the differential equations governing the kinetics of $N_1$, $N_2$ and $N_3$: the populations (electrons / volume) for the three states.

(b) In steady state, find the ratio $N_3/N_1$ in order to determine the condition for population inversion ($N_3 > N_1$). Find and interpret the $I_{\text{pump}} \to \infty$ limit of this criterion.

(c) What is the net power density (rate of energy change per unit volume) absorbed from the pump light into the electrons? Assume that stimulated emission puts energy back into $I_{\text{pump}}$ (best case scenario for efficiency), while the energy from spontaneous emission is lost. Just write the answer in terms of instantaneous $N_1$, $N_2$, $N_3$ (don't solve for the $N$s).

(d) Similarly, what is the power density output from the $3 \to 1$ transition (fluorescence)? Again, just express in terms of $N_1$, $N_2$, $N_3$ as needed.

(e) What is the energy efficiency of the fluorescence process in steady state, and how does it depend on $I_{\text{pump}}$? (This time, solve for the $N$s in terms of the $A, B$ parameters and interpret!)