# HW7 solution

MTLE-6120: Spring 2017

Due: Apr 27, 2017

1. **Kasap 5.29: Seebeck coefficient and thermal drift in semiconductor devices**

   (a) Holes and electrons both diffuse from hot to cold sides, but correspond to opposite charge transport directions. Electrons lead to negative $S$ in $n$-type semiconductors, compared to holes which yield positive $S$ in $p$-type semiconductor.

   (b) Charge neutrality for an $n$-type semiconductor yields $N_c \exp \frac{E_F - E_c}{k_B T} = N_d$. Therefore the Seebeck coefficient is:

   $$S_n = -\frac{k_B}{e}\left[2 + \frac{E_c - E_F}{k_B T}\right] = -\frac{k_B}{e}\left[2 + \ln \frac{N_c}{N_d}\right]$$

   Given that $k_B/e = 8.63 \times 10^{-5}$ V/K and $N_c = 2.8 \times 10^{19}$ cm$^{-3}$, we can easily calculate $S_n = -1.26 \times 10^{-3}$ V/K for $N_d = 10^{14}$ cm$^{-3}$ and $-8.58 \times 10^{-4}$ for $N_d = 10^{16}$ cm$^{-3}$. The implication for devices is that temperature changes and gradients can induce changes in the voltages at the outputs of devices, thereby contributing to thermal drift.

   (c) We have already calculated $S_n = -1.26 \times 10^{-3}$ V/K for $N_d = 10^{14}$ cm$^{-3}$ $n$-type Si. We can neglect $S_p$ because with this asymmetric doping, most of the potential drop is in the $n$ side. The potential corresponding to 1C i.e. 1K change of temperature is 1.26 mV, which amplified by a gain of 100 will produce an output of 0.13 V.

2. **Kasap 6.2: The Si $pn$ junction** (estimating recombination and diffusion currents)

   In answering, 'what is your conclusion', include your expectation for the diode ideality factor $\eta$.

   Hint: you will need the diffusion constants for the carriers. Assume $D_e = 34.9$ cm$^2$/s and $D_h = 11.6$ cm$^2$/s. (Diffusion constants are related to mobilities by the Einstein relation $D_{e/h} = (k_B T/e)\mu_{e/h}$) Besides that, the only other $Si$-specific properties you should need are $n_i = 10^{10}$ cm$^{-3}$ and $\epsilon_r = 11.7$.

   The diffusion current is $j_s = j_{s0} \exp \frac{eV}{k_B T}$ where

   $$j_{s0} = e n_i^2 \left[\frac{D_h}{L_h N_d} + \frac{D_e}{L_e N_a}\right] = e n_i^2 \left[\frac{1}{\sqrt{D_h \tau_h} N_d} + \frac{1}{\sqrt{D_e \tau_e} N_a}\right]$$

   while the recombination current is $j_r = j_{r0} \exp \frac{eV}{2k_B T}$ where

   $$j_{r0} = \frac{e n_i}{2}\left[\frac{w_p}{\tau_e} + \frac{w_n}{\tau_h}\right].$$

   Note that for this we first need to calculate the built-in potential

   $$V_0 = k_B T \log \frac{N_a N_d}{n_i^2},$$

   the depletion width

   $$w_0 = \sqrt{\frac{2\epsilon(N_a + N_d)(V_0 - V)}{e N_a N_d}}$$

   and from that the widths on the two sides $w_p$ and $w_n$.

(a) Let us start with the recombination times. Using the given formula, $\tau_e = 2.4 \times 10^{-8}$ s in the $p$-region (irrelevant) and $\tau_h = 4.9 \times 10^{-7}$ s in the $n$-region (required).

The built in potential evaluates to $V_0 = 0.77$ V, which yields a depletion width of $w_0 = 4.74 \times 10^{-5}$ cm (474 nm). This splits into $w_p = 4.74 \times 10^{-8}$ cm (0.47 nm, irrelevant) and $w_n \approx w_0 = 4.73 \times 10^{-5}$ cm (473 nm, required).

From these, we can calculate the prefactors for the two current contributions as $j_s0 = 6.74 \times 10^{-12}$ A/cm$^2$ and $j_{r0} = 1.58 \times 10^{-7}$ A/cm$^2$. Finally accounting for the area of 1 mm$^2$ = 0.01 cm$^{-2}$ and the exponential factors, we get

$$I_s = 0.82 \text{ mA} \quad \text{and} \quad I_r = 0.17 \text{ mA}$$

Therefore recombination is comparable but smaller than the diffusion contribution, which will yield an ideality coefficient closer to 1.

(b) The recombination times are, $\tau_e = \tau_h = 2.4 \times 10^{-8}$ s on both sides of the interface.

The built in potential evaluates to $V_0 = 0.95$ V, which yields a depletion width of $w_0 = 3.02 \times 10^{-6}$ cm (30 nm). This splits evenly into $w_p = w_n = 1.51 \times 10^{-6}$ cm (15 nm).

From these, we can calculate the prefactors for the two current contributions as $j_s0 = 4.80 \times 10^{-14}$ A/cm$^2$ and $j_{r0} = 2.03 \times 10^{-7}$ A/cm$^2$. Finally accounting for the area of 1 mm$^2$ = 0.01 cm$^{-2}$ and the exponential factors, we get

$$I_s = 5.8\ \mu\text{A} \quad \text{and} \quad I_r = 0.22 \text{ mA}$$

Therefore recombination far exceeds the diffusion contribution, which will yield an ideality factor closer to 2.

3. **Kasap 6.15: Ultimate limits to device performance** (of an $n$-channel FET)

Note: for part (c), assume and justify reasonable values for the barriers. Consider tunneling to become important when tunneling probabilities $\sim 10^{-6}$.

Also, there might be a typo in the example number referenced by some editions of the book. (In mine, it says 3.10, but it should be 3.12.)

(a) The electron PE changes by $eV_{DS} \geq k_BT \approx 0.026$ eV, in order to exceed thermal fluctuations. Therefore $V_{DS} \geq 0.026$ V.

Given a field of $10^6$ V/m, this $V_{DS}$ corresponds to a channel length $L \sim 2.6 \times 10^{-8}$ m (26 nm).

With saturated velocity of $10^5$ m/s, this corresponds to a transit time of $\tau_t \sim 2.6 \times 10^{-13}$ s (0.26 ps).

(b) The characteristic time associated with $eV_{DS} \sim k_BT \approx 0.026$ eV is $\tau \sim \hbar/(k_BT) \sim 2.5 \times 10^{-14}$ s (25 fs), an order of magnitude smaller than the transit time above.

(c) We can assume that the barriers for tunneling are $\sim 1$ eV. This is the typical magnitude for built-in potentials in Si, which affects the barrier for tunneling across the channel. This typical band offsets between Si and SiO$^2$ are also in the same order of magnitude, with $\sim 3$ eV barrier for electrons and $\sim 2$ eV barriers for holes.

The tunneling probability is $T \sim \exp(-2L\sqrt{2m(V-E)}/\hbar)$. For barrier $(V-E) \sim 1$ eV, the wavevector $\kappa = \sqrt{2m(V-E)}/\hbar \sim 5 \times 10^9$ m$^{-1}$. Therefore for $\exp(-2L\kappa) \sim 10^{-6}$, $L \sim 1.4$ nm. At these channel lengths or oxide thicknesses, tunneling will become a limitation.