HW6

MTLE-6120: Spring 2017

Due: Apr 17, 2017

1. **Critical magnetic field of a superconductor**

   Consider a metal with some density of states per unit volume $g(E)$ in its normal state, which becomes a BCS superconductor below a critical temperature T_c. In BCS theory, the superconducting gap at zero temperature is given by $\Delta = 1.76 k_B T_c$.

   (a) If all the electrons with energy $E_F - \Delta < E < E_F$ pair up with binding energy Δ per pair, what is the gain in energy density of the superconductor relative to the normal metal?

   (b) What is the energy density incurred in expelling a magnetic field B due to the Meissner effect?

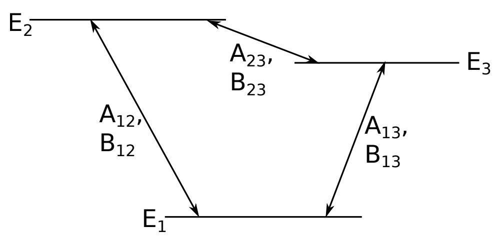
   (c) Given that at the critical magnetic field B_c, it is no longer energetically favorable to expel the magnetic field, relate B_c to Δ (at $T = 0$).

   (d) Aluminum is face-centered cubic metal with a cubic lattice constant of 4.05 Å, which behaves almost perfectly like a free-electron metal with 3 free electrons per atom. What is its $g(E_F)$ in SI units ($J^{-1}m^{-3}$)?

   (e) Given that aluminum becomes a BCS superconductor below $T_c = 1.2$ K, estimate its zero-temperature critical magnetic field B_c in SI units (Tesla).

2. **Optical pumping and fluorescence**

   Cosnider the minimal three-level system necessary for fluorescence (and lasing) as shown below. A pump light of intensity I_{pump} is tuned to a frequency matching $E_2 - E_1$, and assume the intensities at other frequencies are small enough that stimulated emission is negligible for the other transitions. Assume A and B coefficients for each pair of states as shown.



   (a) Write the differential equations governing the kinetics of N_1, N_2 and N_3: the populations (electrons / volume) for the three states.

   (b) In steady state, find the ratio N_3/N_1 in order to determine the condiction for population inversion ($N_3 > N_1$). Find and interpret the $I_{\text{pump}} \to \infty$ limit of this criterion.

   (c) What is the net power density (rate of energy change per unit volume) absorbed from the pump light into the electrons? Assume that stimulated emission puts energy back into I_{pump} (best case scenario for efficiency), while the energy from spontaneous emission is lost. Just write the answer in terms of instantaneous N_1, N_2, N_3 (don't solve for the Ns).

   (d) Similarly, what is the power density output from the $3 \to 1$ transition (fluorescence)? Again, just express in terms of N_1, N_2, N_3 as needed.

   (e) What is the energy efficiency of the fluorescence process in steady state, and how does it depend on I_{pump}? (This time, solve for the Ns in terms of the A, B parameters and interpret!)