HW5 solution

MTLE-6120: Spring 2017

Due: Apr 6, 2017

1. **Kasap 7.20: Capacitor design**

   (a) For low voltage application, one can use the minimum thickness to minimize the volume. The capacitance is then $C = \epsilon_r \epsilon_0 A/d_{\min} = \epsilon_r \epsilon_0 V/d_{\min}^2$, so that the volume is $C d_{\min}^2/(\epsilon_0 \epsilon_r)$. Substituting from table 7.13

	PET	TiO_2	$BaTiO_3$
Volume [m^3]	$(3.5 - 14) \times 10^{-9}$	1.26×10^{-8}	6.3×10^{-10}

   Note the significant reduction in volume achievable with high dielectric materials, even with a higher minimum thickness. This makes $BaTiO_3$ produce the lowest volume.

   (b) For high voltage applications, the thickness may be limited by the breakdown field. We now require $V_{\max}/d < \mathcal{E}_{\text{br}}/2$ i.e. $d = 2V_{\max}/\mathcal{E}_{\text{br}}$

	PET	TiO_2	$BaTiO_3$
d [μm]	67	200	100
Volume [m^3]	1.6×10^{-5}	5×10^{-6}	6.3×10^{-8}

   The high dielectric constant of $BaTiO_3$ still wins in minimizing the volume, and by an even larger margin because breakdown limits the thickness of PET by a larger factor than it does the remaining two.

   (c) The dissipated power is $P = CV^2\omega \tan\delta = 2\pi V^2 \nu \tan\delta$. Substituting $C = 100$ nF and $\nu = 60$ Hz:

	PET	TiO_2	$BaTiO_3$
P [W]	0.047	0.0038	0.47

   Note that in case the power dissipation is determined entirely by the loss tangent $\tan\delta$, since the other parameters are constant across materials for the same capacitance and operating conditions. The lowest dissipation is therefore for the lowest loss tangent material, TiO_2.

2. **Kasap 8.6: Ferromagnetism and the exchange interaction**

   Note that there are effectively three parts to this question, as listed below for clarity:

   (a) What is the spin magnetic moment of the isolated Dy atom based on its electronic configuration?
   The spin magnetic moment is due to $4f^{10}$, which given that there are seven f orbitals, corresponds to ↑↓, ↑↓, ↑↓, ↑, ↑, ↑, ↑, and therefore four unpaired spins. The corresponding magnetic moment is $4 \times g\mu_B/2 \approx 4\mu_B$.

   (b) What is the magnetic moment per Dy atom in the solid based on the saturation magnetization? Compare with (a).
   From the density and atomic mass, the number density of atoms is (8.54 g/cm^3) / (162.50 g/mol) $\times$ 6.022×10^{23}/mol $= 3.16 \times 10^{22}$/cm^3. The saturation magnetization per unit volume is 2.4×10^6 A/m, which corresponds to $(2.4 \times 10^6 \text{ A/m})/(3.16 \times 10^{22}/\text{cm}^3) = 7.6 \times 10^{-23}$ Am^2 per atom.

Since the Bohr magneton is 9.274×10^{-24} Am^2, this magnetization corresponds to 8.2 μ_B or 8.2 spins per atom.

Therefore the magnetization per atom in the solid is twice that in the isolated atom. This is perfectly reasonable because in metals, the number of unpaired spins depends on where the Fermi level is relative to the band density of states. This can differ considerably form the atomic picture.

(c) Estimate the exchange interaction magnitude given the Curie temperature.

Assuming $E_{ex} \sim k_B T_c$ with $T_c \sim 85$ K yields $E_{ex} \sim 7.4 \times 10^{-3}$ eV.

3. **Kasap 8.17: Superconductivity and critical current density**

(a) The current in Sn when the surface field reaches the critical value is $I_{\max} = 2\pi r B/\mu_0 = 2\pi(5 \times 10^{-4})(0.2)/(4\pi \times 10^{-7}) = 500$ A. The corresponding current density is $j_{\max} = (500 \text{ A})/(\pi(5 \times 10^{-4} \text{ m})^2) = 6.4 \times 10^8$ A/m^2.

(b) Correspondingly for $B_{c2} = 24.5$ T in Nb_3Sn, $I_{\max} = 6.1 \times 10^4$ A and $j_{\max} = 7.810$ A/m^2.

This maximum current density is close to the critical $j_c = 10^{11}$ A/m^2, but that is not always true. Remember that the $I_{\max}$ and $j_{\max}$ depend on the wire diameter, whereas j_c is a geometry-independent material property.