HW2

MTLE-6120: Spring 2017

Due: Feb 13, 2017

1. **Photoelectric effect and quantum efficiency: Kasap 3.8**

2. **Scattering by a δ-atom**

   (a) Consider an electron with energy $E = \frac{\hbar^2 k^2}{2m} > 0$ incident from the left ($-\infty$) towards a δ-atom given by the potential $-V_0\delta(x)$, with strength parametrized using the parameter $q = 2mV_0/\hbar^2$. Find the wavefunction $\psi(x)$ (expressed in k and q, rather than E and V_0 for convenience). Can you normalize it?

   (b) What are the probabilities of the electron getting reflected and transmitted? Describe and explain their qualitative dependence on the energy of the electron. What would the corresponding probabilities be in the classical case?

   (c) Consider the opposite potential $+V_0\delta(x)$. What is the electron wavefunction? What are the probabilities of the electron getting reflected and transmitted? What would the corresponding probabilities be in the classical case?

   (d) Instead of a free electron with definite momentum k (and energy $E = \frac{\hbar^2 k^2}{2m}$) as considered above, consider a Gaussian wavepacket with coefficients

$$c_k = \frac{1}{\sqrt{\sigma_k\sqrt{2\pi}}} \exp\frac{-(k-k_0)^2}{2\sigma_k^2},$$

   assuming that the spread in momentum $\sigma_k \ll k_0$ and q. Where is the electron as a function of time?

3. **Photoemission from a δ-atom**

   The δ-atom with potential $-V_0\delta(x)$ has a single bound state $e^{-\kappa|x|}\sqrt{\kappa}$ with $\kappa = mV_0/\hbar^2 \equiv q/2$ (as shown in class), and free states at any energy $E > 0$ (previous problem). Now apply a time-varying electric field $\mathcal{E}e^{-i\omega t}$ in the x direction (from an EM wave, for example).

   (a) For the atom in its ground state, what is the minimum ω that can free the electron?

   (b) For the free states solved in the previous problem, let us solve the normalization problem by assuming that the entire system is inside an infinite potential well of length L (centered at $x = 0$). What are the normalized wavefunctions and the condition(s) satisfied by the allowed k? (Hint: there are two classes of solutions: odd and even about $x = 0$. Also, you may not be able to solve for k explicitly in some cases; only write the condition satisfied by k in those cases.)

   (c) What are the matrix elements $\langle\psi_f|H'|\psi_i\rangle$ of the perturbation Hamiltonian $H' = -e\mathcal{E}x$ between the bound state (initial) and the free states (final) determined above? Use $L \gg 1/k, 1/\kappa$ to simplify the integrals. (We are going to take $L \to \infty$ below.) Are there any selection rules?

   (d) Using Fermi's Golden rule, calculate the rate of photo-excitation out of the bound state. This requires summing over the final states which have separation in k that is proportional to $1/L$. Take the limit $L \to \infty$ so that this becomes an integral. Describe the frequency dependence of the photoemission rate.